Public Sector Wage Policy and Labor Market Equilibrium: A Structural Model

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Abstract

We develop and estimate a structural model that incorporates a sizable public sector in a labor market with search frictions. The wage distribution and the employment rate in the public sector are taken as exogenous policy parameters. Overall wage distribution and employment rate are determined within the model, taking into account the private sector's endogenous response to public sector employment policies. Job turnover is sector specific and transitions between sectors depend on the worker's decision to accept alternative employment in the same or different sector by comparing the value of employment in the current and prospective jobs. The model is estimated on British data by a method of moments. We use the model to simulate the impact of various counterfactual public sector employment policies.

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1 Introduction

We formulate a search-theoretic model that incorporates interactions between the public and private sectors. The wage offer distribution of the public sector is treated as an exogenous policy parameter, and conditional on this and exogenous transitional parameters, the offer and observed private sector wage distributions are then derived endogenously. Exploiting data from the British Household Panel Survey (BHPS) the model is then estimated by minimum-distance matching of some key moments from the data. These estimates allow us to make counterfactual policy analysis of different public sector wage policies.

There has been very little done in modeling the public sector explicitly within an equilibrium model of the labor market and nothing to our knowledge that estimates such a model. This is a considerably large oversight when one thinks that in our data 24% of employed individuals were employed by the public sector.[1] It is, of course, naive to believe that with an employment share this large the public sector will not influence wage determination and by extension overall employment.

Instead of modeling the behavior of private sector firms explicitly, the literature thus far has been dominated by reduced form comparisons of the two sectors.[2] The general consensus of stylized facts emerging from the empirical literature is that the public sector wage distribution is more compressed than the private sector and workers receive a small public sector wage premium which is more prevalent in low skilled workers.

With these stylized facts being known for some time, it is fairly surprising that so little has been done in explicitly modeling the interaction between the two sectors. The existing literature that does this has largely focused on assessing the impact of the public sector on the level or volatility of aggregate wages and employment. Papers in that vein include Algan et al. (2002), Quadrini and Trigari (2008), Hörner et al. (2007), and Gomes (2011). All model search as directed to a particular sector and none has direct job-to-job reallocation (within or between sectors). All model wages as determined by bargaining over the surplus from a match. Algan et al. (2002) find that the creation

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[1]Algan et al. (2002) report that, based on a slightly narrower definition of public sector employment, the OECD finds an average public sector share of total employment of 18.8% in 2000 over a sample of 17 major OECD countries.

[2]For a survey of the literature, see Bender (1998).

of public sector jobs has a massive crowding out effect on private-sector job creation, such that the marginal public sector job may destroy as many as 1.5 private sector jobs in some OECD countries. This crowding out effect is especially strong when public sector wages are high and/or when public and private-sector output are close substitutes. Focusing on the cyclicality of employment, Quadrini and Trigari (2008) examine a public sector wage policy that is acyclical (a single wage) and pro-cyclical (government wage is an increasing function of private sector wages). Calibrating the model for the US economy, they predict that the volatility of employment increased by two and four times, with the existence of the public sector over the periods 1970-2003 and 1945-1970, respectively. They postulate that this reduction over time is because of a more pro-cyclical policy adopted by the state. Hörner et al. (2007) model two economies: one where a benevolent social planner aims to maximize individual's welfare with public sector wages and employment (amongst other matters); the other in the absence of a public sector. The equilibrium of the model allows the authors to draw two conclusions. Firstly, that the public sector has an ambiguous effect on overall employment and secondly, that in more turbulent times there will be higher unemployment in the economy with the public sector. The latter result comes from the individuals being risk averse and therefore crowding into the safer sector (the public sector) in more uncertain times. Finally, Gomes (2011) builds a dynamic stochastic general equilibrium model with search and matching frictions calibrated to U.S. data and shows that high public sector wages induce too many unemployed to apply for public sector jobs and raise unemployment. He further argues that the cyclicality of public sector wage policy has a strong impact on unemployment volatility.

To our knowledge, other than this paper, the extent of the literature that explicitly models private sector firm behavior for a given public wage setting policy is Albrecht et al. (2011) and Burdett (2011). Albrecht et al. (2011) extend the canonical Diamond-Mortensen-Pissarides model (Pissarides, 2000) to incorporate the public sector posting an exogenous number of job vacancies. They also introduce match-specific productivity and heterogeneity in worker's human capital, which generate wage dispersion in the private sector. Burdett (2011) is closer to ours in the sense that firms post wages rather than bargain over the surplus and, crucially, that workers are allowed to search

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on the job. On-the-job search — and the possibility for workers to switch jobs between sectors — is an essential ingredient of the model as it captures rich and complex aspects of the competition between sectors over the labor services of workers. Moreover, as is understood since at least Burdett and Mortensen (1998), on-the-job search begets endogenous wage dispersion. However, in Burdett's model the public sector sets a single wage, leading to the counterfactual prediction that the private sector's response to competition from the public sector is to post a wage distribution with a hole in its support. In our model the public sector's policy is to post wages from a distribution of wages. This allows us to have wage differences in the public sector and a continuous private sector wage distribution with connected support. Unlike the two models discussed we allow for differences in cross-sector job destruction and job offer arrival rates. Crucially, this paper is unique in the literature insofar as the parameters of the model are estimated.

Methodologically, a similar paper to ours is Meghir et al. (2010) who develop an equilibrium wage posting model to determine the interaction between a formal and informal sector in a developing country. Here the two sectors vary in the degree of regulatory tightness, the formal sector firms incurring additional costs to wages in the form of corporation tax, income tax, social security contributions, severance pay and unemployment insurance. While firms in the informal sector are not exposed to these labor market regulations they do face the chance (with given probability) of incurring a non-compliance cost. Unlike this model, private sector firms endogenously select into either the formal or informal sector and the equilibrium wage offer distributions of both are determined endogenously, similarly to this paper, Meghir et al. estimate their model using indirect inference.

The paper is organized as follows. In the next section we derive the equilibrium structural model. Section 3 gives an overview of the properties of the data used for estimation. Sections 4 and 5 outline the estimation protocol and present the estimation results. In Section 6 we use the results obtained to run counterfactual policy analysis and in Section 7 we conclude.

2 The Model

2.1 Basic Environment

We consider a model of wage-posting akin to Burdett and Mortensen (1998). Time is continuous and the economy is in steady state. A $[0, N]$ continuum of infinitely lived, risk neutral, ex-ante homogeneous workers face a fixed continuum of employers in a frictional labor market. A key aspect of our approach is that the set of employers comprises a continuum of infinitesimally small hetero-geneous, profit-maximizing firms which we interpret as Private Sector employers, that coexist with a single, non-infinitesimal, non-profit maximizing employer which we interpret as the Public Sector. Private-sector firms behave in the same way as employers in the standard Burdett and Mortensen (1998) model, while public sector wages and level of labor demand are taken as exogenous. Because of the public sector's non-infinitesimal size as an employer, changes in public sector employment policies will have a non-trivial impact on labor market equilibrium, both directly and through the private sector's response to said changes in policy. The main objective of this paper is to quantify that impact for various policy changes.

2.2 Workers and Jobs

A worker can be in one of three states, either unemployed or employed in the public or private sector. Throughout the paper we indicate a worker's labor market state using a subscript $s \in \{u, p, g\}$ for unemployment, employment in the private sector and employment in the public sector, respectively. The steady-state numbers of workers in each employment state are denoted as N_u, N_g and N_p.

A job is fully characterized by a constant wage w and the sector it is attached to. Workers receive job offers at a Poisson rate that depends on the worker's state. Jobs are not indefinite and also face a Poisson destruction shock. The notation used to describe all those shocks is largely consistent with the previous literature, δ being used to denote job destruction shocks and λ for job offer arrival rates. To explain the two states between which the particular worker transits a two letter index is used. The first letter designates the sector of origin and the second the sector of destination. So for example, λ_{pg} is the arrival rate of public-sector offers to private sector employees, λ_{ug} is arrival

rate of public-sector offers to unemployed workers, and so on. As job destruction always results in the worker becoming unemployed, a single index is used to specify the job destruction shock, δ_p or δ_g.

To summarize, a worker employed in sector $s \in \{p, g\}$ faces three random shocks: a job destruction shock δ_s, after which the worker becomes unemployed and gets flow utility b, a within sector job offer λ_{ss} and a cross-sector job offer $\lambda_{ss'}$. While the arrival rates from the public sector are exogenous, arrival rates from the private sector and the associated acceptance decisions are determined endogenously.

As in Burdett and Mortensen (1998), a job offer from the private sector consists of a draw from a wage offer distribution $F_p(\cdot)$ which results from uncoordinated wage posting by the set of infinitesimally small private employers, each maximizing its profit taking as given the strategies of other firms and that of the public sector. $F_p(\cdot)$ will be determined endogenously in equilibrium. By contrast, a job offer from the public sector consists of a draw from a (continuous) wage offer distribution $F_g(\cdot)$ which is taken as an exogenous policy tool. We thus assume from the outset that the public sector offers jobs at different wage levels to observationally similar workers. This assumption is both realistic — residual wage dispersion among similar public-sector employees is observed in the data — and needed to avoid the counterfactual prediction of Burdett (2011) that equilibrium features a wage distribution with disconnected support.

We finally recognize that public and private sector jobs may differ along other dimensions than just the wage and the transition parameters. There may be, for example, systematic differences in working conditions. Also workers may enjoy a utility surplus ('public service glow') or suffer a utility loss ('public service stigma') from working in the public sector. In order to capture those unobserved differences in a parsimonious way, we assume that the flow utility that workers derive from working in sector $s \in \{p, g\}$ for a wage of w is equal to $w + a_s$, where a_s is a sector-specific 'amenity'. Finally, and without further loss of generality, we normalize a_p to zero, so that a_g

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reflects the relative utility surplus (or loss) from working in the public sector. This utility surplus is assumed to be the same for all workers.

2.2.1 Worker Values and Reservation Wages

An individual's utility is given by the present discounted future stream of wages. For a given worker, the transitional parameters will be unchanged if he moves job within a sector. The acceptance decision for an offer within the worker's current sector is therefore entirely determined by the worker's current wage and the new wage being offered. If the new offer, x, is higher than the worker's current wage, w, he will accept and otherwise reject. However, since a change in sector is not only associated with a different wage but also with a change in transitional parameters, the acceptance decision is not so trivial when the job offer is from another sector. Thus depending on the two sets of transition parameters, an individual may accept a job offer from a different sector with a wage cut, or conversely, require a higher wage in order to accept. These acceptance decisions can be characterized by a set of reservation wages, accepting job offers greater or equal to these wages and rejecting those below. With the three states we have defined, there will be four corresponding reservation wages, which we define using the same double-index system as for transition parameters: R_{up}, R_{ug}, $R_{pg}(w)$ and $R_{gp}(w)$.

When employed, a worker's reservation wage will be a function of their current wage. The reservation wage applying to private (public) sector offers made to a public (private) sector worker earning w makes said worker indifferent between his current present value and the present value of private (public) sector employment at his reservation wage. Formally, that is $W_p(R_{gp}(w)) = W_g(w)$ and $W_g(R_{pg}(w)) = W_p(w)$, where $W_p(w)$ and $W_g(w)$ are the values of working in the private and public sectors at wage w. It follows from those definitions that the two reservation wages described are reciprocal of each other:

$$R_{pg}(R_{gp}(w)) = w. \tag{1}$$

The reservation wage of an unemployed worker receiving an offer from the public (private) sector is the wage at which they are indifferent between unemployment and the public (private) sector. Formally, the two reservation wages solve the equality, $U = W_p(R_{up}) = W_g(R_{ug})$, where

U is the present value of a worker in unemployment. Hence applying (1) to this equality one can derive a second property of the reservation wages:

$$R_{pg}\left(R_{up}\right) = R_{ug}. \tag{2}$$

Note that the analogous property for $R_{gp}\left(\cdot\right)$ also holds.

2.2.2 Bellman Equations

The value function for an unemployed worker is defined by the following Bellman equation, where r is the rate of time preference, constant across workers:

$$rU = b + \lambda_{up} \int_{R_{up}}^{+\infty} \left[W_p\left(x\right) - U\right] dF_p\left(x\right) + \lambda_{ug} \int_{R_{ug}}^{+\infty} \left[W_g\left(x\right) - U\right] dF_g\left(x\right), \tag{3}$$

The first term, b is the flow utility an individual gets from being in unemployment. Offers arrive from the public (private) sector at a rate of λ_{up} (λ_{up}). Wage offers, x are drawn from the private sector from an endogenous distribution, $F_p\left(w\right)$, which will be derived from the firm side later. An unemployed worker will accept the job offer if the wage is higher than the worker's reservation wage for that sector, the lower bound of the integral. Inside the integral is the gain the worker makes from switching from unemployment to private sector employment at wage w. The final term is the public sector analogue to the second. The theoretical difference between the two is that the distribution from which public-sector job offers are drawn is an exogenous policy parameter of the model.

Similar value functions define a worker employed in the private and public sectors. Below is the example for a private sector employee:

$$rW_p\left(w\right) = w + \delta_p \left\{U - W_p\left(w\right)\right\}$$
$$+ \lambda_{pp} \int_{w}^{+\infty} \left[W_p\left(x\right) - W_p\left(w\right)\right] dF_p\left(x\right) + \lambda_{pg} \int_{R_{pg}(w)}^{+\infty} \left[W_g\left(x\right) - W_p\left(w\right)\right] dF_g\left(x\right) \tag{4}$$

A worker employed in the private sector and earning a wage w has a discounted value from employment given by the right hand side of (4). The first term w is the instantaneous wage paid in the current private sector firm. The next term, is the loss of value an individual would get if he

were to transit into unemployment $[U - W_p(w)]$ multiplied by the flow probability of such an event occurring, the private sector job destruction rate, δ_p. At rate λ_{pp} the worker receives an offer from another private sector firm, where the offer is drawn from the distribution $F_p(x)$. If this offer is greater than his current wage w he will accept, the lower bound of the integral. Given the offer is received and it meets his acceptance criteria, the individual will make an unambiguous gain in value given by $[W_p(x) - W_p(w)]$. The next term represents the equivalent, except for offers from the public sector. Thus the wage is drawn from a different distribution and the acceptance criteria, the lower bound of the integral is instead $R_{pg}(w)$. An analogous Bellman equation defines the value function for a worker in the public sector:

$$rW_g(w) = w + a_g + \delta_g \{U - W_g(w)\}$$
$$+ \lambda_{gp} \int_{R_{gp}(w)}^{+\infty} [W_p(x) - W_g(w)] \, dF_p(x) + \lambda_{gg} \int_w^{+\infty} [W_g(x) - W_g(w)] \, dF_g(x). \quad (5)$$

Note the presence of the additional flow utility term a_g, the 'public-sector amenity' discussed above.

The value functions given by (3), (4) and (5) allow us to obtain the reservation wage required to leave the private for the public sector and vice-versa as a function of the transition parameters. This is done using the identity $W^p(R_{pg}(w)) = W_g(w)$ and $W_g(R_{gp}(w)) = W_p(w)$ and assuming differentiability of the value functions. This manipulation is performed in Appendix A and the solution for a private sector worker's reservation wage from the public sector solves the following non-linear ODE:[3]

$$R'_{pg}(w) = \frac{r + \delta_g + \lambda_{gp}\overline{F}_p(w) + \lambda_{gg}\overline{F}_g(R_{pg}(w))}{r + \delta_p + \lambda_{pp}\overline{F}_p(w) + \lambda_{pg}\overline{F}_g(R_{pg}(w))}, \quad (6)$$

with initial condition $R_{pg}(R_{up}) = R_{ug}$. It should be noted that R_{up} and R_{ug} themselves depend on the functions $R_{pg}(\cdot)$ and $R_{gp}(\cdot)$ as they are obtained by solving $W_s(R_{us}) = U$ for $s = p$ or g. However, they also depend on the bs, which are free parameters, so those reservation wages can themselves be estimated as free parameters.

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[3]Here and throughout the rest of the paper, a bar over a c.d.f. denotes the survivor function, so for example $\overline{F}_p(\cdot) := 1 - F_p(\cdot)$.

2.2.3 Flow-Balance Equations and Worker Stocks

The economy being in steady-state, the flows in and out of any given sector, for each class of workers, are equal. Applying this to unemployment, one obtains:

$$(\lambda_{up} + \lambda_{ug}) N_u = \delta_p N_p + \delta_g N_g \tag{7}$$

The left hand side of (7) is the rate at which workers leave unemployment toward the two sectors of employment. This occurs when a worker receives a job offer from a given employment sector and the associated wage offer is higher than his appropriate reservation wage. Assuming homogeneous workers, there is no reason why a firm would offer wages below a worker's reservation wage (and if it did, it would employ no worker and therefore become irrelevant to market equilibrium). Therefore we assume without (further) loss of generality that $F_p(R_{up})$ and $F_g(R_{ug})$ are equal to zero. The right hand side is at the unemployment inflow, which consists of workers being hit by job destruction shocks in their sector of employment. A worker can only be in one of three states, u,p or g so: $N_u + N_p + N_g = N$, where N is the total population of workers, a given number.

Equation (8) is the flow-balance equation for private sector workers, equating the flow into the private sector below a wage w to the flow out, thus imposing that not only is the share of private sector workers static in our model so is the distribution of wages amongst them. The left hand side is the flow out of private employment. $N_p G_p(w)$ is the number of private sector workers earning less than a wage w. They can exit to unemployment through job destruction shocks δ_p. The second and third terms are the exit rates into the public sector and higher paid private sector jobs, respectively, upon receiving a job offer (λ_{pg} and λ_{pp}). The right hand side is the flow into private sector employment, the first term being the flow from unemployment and the second, from the public sector.

$$N_p \delta_p G_p(w) + N_p \lambda_{pg} \int_{R_{up}}^{w} \overline{F}_g(R_{pg}(x))\, dG_p(x) + N_p \lambda_{pp} \overline{F}_p(w) G_p(w)$$
$$= N_u \lambda_{up} F_p(w) + N_g \lambda_{gp} \int_{R_{ug}}^{R_{pg}(w)} [F_p(w) - F_p(R_{gp}(x))]\, dG_g(x) \tag{8}$$

Rearranging equation (8) and differentiating with respect to the wage rate, w, one obtains:

$$\frac{d}{dw}\left\{\left[\delta_p + \lambda_{pp}\overline{F}_p\left(w\right)\right]N_pG_p\left(w\right)\right\} + N_pg_p\left(w\right)\lambda_{pg}\overline{F}_g\left(R_{pg}\left(w\right)\right)$$

$$- N_g\lambda_{gp}G_g\left(R_{pg}\left(w\right)\right)f_p\left(w\right) = N_u\lambda_{up}f_p\left(w\right). \quad (9)$$

This would be a fairly straightforward ODE in $g_p\left(w\right)$ (the probability distribution of observed wages in the private sector), if it was not for the term featuring $G_g\left(R_{pg}\left(w\right)\right)$. This term can be derived by manipulation of the flow balance equation for public sector workers earning less than $R_{pg}(w)$ (instead of w). This manipulation is performed in Appendix A. Plugging this solution into (9), we obtain an ODE that defines $G_p\left(w\right)$.

An additional hurdle at this point is the determination of N_p and N_g (with $N_u = N - N_p - N_g$). Those numbers are needed to solve for $G_p\left(\cdot\right)$ in the ODE resulting from the combination of (9) and the isolation of $N_gG_p\left(R_{pg}(w)\right)$, given in the appendix. Now, N_p and N_g are jointly defined by the balance of flows in and out of employment (7), and the flow balance in and out of, say, the private sector, which is given by evaluating the flow-balance equation of private sector workers, equation (8) at $w \rightarrow +\infty$:

$$N_p\delta_p + N_p\lambda_{pg}\int_{R_{up}}^{+\infty}\overline{F}_g\left(R_{pg}\left(x\right)\right)dG_p\left(x\right)$$

$$= N_g\lambda_{gp}\int_{R_{ug}}^{+\infty}\overline{F}_p\left(R_{gp}\left(x\right)\right)dG_g\left(x\right) + N_u\lambda_{up}\overline{F}_p\left(R_{up}\right) \quad (10)$$

The distribution, $G_g\left(\cdot\right)$, can be derived using the identity $R_{pg}\left(R_{gp}\left(w\right)\right) = w$ applied to the derivation of $G_p\left(R_{pg}(w)\right)$ in the appendix. The latter equation involves $G_p\left(\cdot\right)$, which in turns depends on N_p and N_g, so that those three objects have to be solved for simultaneously. This will be done using an iterative procedure.

2.3 Private Sector Firms

There exists a [0,1] continuum of private sector firms who are profit maximizers and heterogeneous in their level of productivity, y, where $y \sim \Gamma\left(\cdot\right)$ over the support $[y_{\min}, y_{\max}]$ in the population of firms. Firms set their wage w and their search effort (number of vacancies, advertising, recruitment

agencies) in order to make a number of hires h. The pair (w, h) is chosen so as to maximize steady-state profit flow. A private sector firm choosing to pay w will experience a quit rate of $\Delta(w)$ of its employees, where:

$$\Delta(w) = \delta_p + \lambda_{pp}\overline{F}_p(w) + \lambda_{pg}\overline{F}_g(R_{pg}(w))$$

As a consequence, the steady-state size of this firm will be $\ell(w, m)$:

$$\ell(w, h) = \frac{h}{\Delta(w)} \tag{11}$$

and its steady-state profit flow:

$$\Pi(w, h) = (y - w)\,\ell(w, h) - c(h) = (y - w)\frac{h}{\Delta(w)} - c(h) \tag{12}$$

where $c(h)$ is the cost incurred by the firm to make h hires. Optimal wage and search policies $w^\star(y)$ and $h^\star(y)$ can thus be characterized using the following first-order conditions:

$$y = w^\star - \frac{\Delta(w^\star)}{\Delta'(w^\star)} \tag{13}$$

$$c'(h^\star) = \frac{-1}{\Delta'(w^\star)} \tag{14}$$

The proportion of all offers that are accepted from a private sector firm offering a wage w is $\alpha(w)$.

$$\alpha(w) = \frac{\lambda_{up}N_u + \lambda_{pp}N_pG_p(w) + \lambda_{gp}N_gG_g(R_{pg}(w))}{\lambda_{up}N_u + \lambda_{pp}N_p + \lambda_{gp}N_g}$$

It follows that the total number of contacts in the economy is the number of hires divided by the acceptance rate:

$$M = \int_{y_{\min}}^{y_{\max}} \frac{h^\star(y)}{\alpha(w^\star(y))}d\Gamma(y), \tag{15}$$

and that the fraction of these contacts that is attached to wage lower than a given w, in other words the probability that a wage offer is less than w can be written in the two following manners:

$$F_p(w) = \frac{1}{M}\int_{y_{\min}}^{\tilde{y}} \frac{h^\star(y)}{\alpha(w^\star(y))}d\Gamma(y), \tag{16}$$

where $\tilde{y}$ is such that $w^\star(\tilde{y}) = w$.

Similarly, the fraction of employees earning a wage less than $w^\star(y)$ do so because they are employed by firms with a productivity lower than y. Thus:

$$H\left[\ell\left(w^\star(y), h^\star(y)\right)\right] = G_p\left[w^\star(y)\right], \tag{17}$$

where $H(\cdot)$ is the distribution of firm sizes among employed workers.

We are now in a position to close the model given public sector policy choices and endogenize private sector job offer arrival rates. To this end, we need to make one final assumption –that the relative search intensities of workers in the three labor market states, i.e. unemployment, employment in the private sector and employment in the public sector are constant. These will be denoted s_{up} (normalized to 1 without loss of generality), s_{pp} and s_{gp} respectively. The arrival rates of private sector offers hence have the following expressions:

$$\lambda_{up} = \lambda_p$$

$$\lambda_{pp} = s_{pp} \cdot \lambda_p$$

$$\lambda_{gp} = s_{gp} \cdot \lambda_p.$$

The private sector job offer arrival rate λ_p (per search efficiency unit) can then be recovered from the following and equation (15), for a given distribution of productivities in the population of firms $\Gamma(y)$:

$$M = \lambda_p\left(N_u + s_{pp}N_p + s_{gp}N_g\right). \tag{18}$$

This and equation (16) illustrate the private sector firms' response to changes in public sector policy in terms of search effort (or number of offers) and wage offer distribution, determining λ_p and $F_p(\cdot)$ respectively, given public sector hiring policy, embodied in the $\{\lambda_{ug}, \lambda_{pg}, \lambda_{gg}\}$ rates, wage offer policy, embodied in the distribution $F_g(\cdot)$ and "job security" policy, embodied in the public sector layoff rate δ_g. Note however that we do not consider any response of the private sector in terms of its layoff rate δ_p as this is considered exogenous and unresponsive to labor market changes.

3   Data and Estimation

We now outline our estimation protocol, which is based on minimum-distance matching of certain descriptive moments of the data.[4] We set the discount rate r ex-ante at 0.004 (where one unit of time is a month), implying an annual rate of approximately 5%. θ, given below is the exogenous parameter vector which we intend to estimate.

$$\theta = (b, a_g, \delta_p, \delta_g, \lambda_{up}, \lambda_{ug}, \lambda_{pp}, \lambda_{pg}, \lambda_{gp}, \lambda_{gg}, F_p, F_g, \Gamma, c(\cdot))'$$

Note that the two offer distributions, the distribution of firm types (Γ) and the cost function of making h hires $c(h)$ feature in the list of parameters. As we will argue below, those distributions are non-parametrically identified. However, for numerical tractability, we will make parametric assumptions on F_p and F_g as outlined later.

The rest of this section focuses on obtaining estimates for the vector θ. We begin by describing the moments we match and how we obtain them from the data, we then describe in detail the estimation procedure. Results are presented in the next section.

3.1   The Sample

The data used in the analysis are taken from the BHPS, a longitudinal data set of British households. Data were first collected in 1991 and the households selected were determined by an equal probability sampling mechanism.[5] Since then, there have been 18 further waves, collected annually. The model outlined is derived under a steady state assumption. Therefore it is necessary that the time period used is short and has approximately constant shares in each of the three states across time. We choose data from 2004 to 2008 to satisfy this assumption, allowing long enough time after the Conservatives' drive toward privatization in the 80s and 90s but before the great recession of 2008.

Using retrospective accounts of employment history we construct a panel dataset of respondents

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[4]For a comprehensive overview of related simulation-based methods, see Gouriéroux et al. (1993).

[5]From wave 9 the BHPS was extended to include Scotland and Wales and from wave 11, Northern Ireland. All three regions are over represented in the sample and therefore we weight the data accordingly.

at a monthly frequency. We include in our data those who across our panel reach at least 21 years of age and don't exceed 60. Income is adjusted for inflation according to the consumer price index and we trim the income distributions in each sector, treating data as missing if it is below the 2nd or above the 98th percentile in the distribution of wage in either employment sector. We also exclude individuals with holes in their employment history and once someone becomes inactive they are from then on excluded. Thus, consistent with our model, an agent can be in one of three states, unemployment or employment in the private or public sectors. We define private sector employment as anyone who declares themselves as employed in a private sector firm, non-profit organization or in self-employment and public sector employment as in the civil service, central or local government, the NHS, higher education, a nationalized industry, the armed forces or a government training scheme. Finally, we restrict our sample to men, to avoid issues of labor supply that are more prevalent amongst women, particularly part-time work and inactivity.

The two sectors vary in their composition of workers; particularly in gender and human capital (see Table 1). We therefore divide our sample into two strata, those who have acquired A-levels and those who haven't. A-levels are taken by 18 year-old British students at the end of further education. We provide estimates separately for the two subsamples. Our sample contains 1,307 high-skill and 1,083 low skill males who we follow for a maximum of 5 years. There is some attrition which we assume to be exogenous. Table 1 shows some basic descriptive statistics. Consistent with the literature on the public-private sector relationship, we find the British public sector is better educated, on average receive higher wages for which there is less dispersion within the sector. Note the proportion of public sector workers appears low as women are over represented in the public sector.

Table 2 conveys information about the extent of job mobility, both within and between sectors. Our data is an unbalanced monthly panel of workers, starting from the first interview date and ending with the last. Counting in each month the number of people making each type of transition and the number in each state, we construct monthly cross-sector transition matrices. Averaging these across our time period, we obtain the transition matrix shown in Table 2.

Table 1: Descriptive Statistics

	Private Sector	Public Sector	Unemployment
size of each sector:			
all	79.64 %	14.73%	5.63%
A-levels and above	75.98%	19.00%	5.02%
less than A-levels	83.67%	10.01%	6.32%
mean hourly earnings			
all	12.42	14.15	-
A-levels and above	14.98	16.03	-
less than A-levels	10.40	11.04	-
standard deviation of hourly wages:			
all	10.53	9.32	-
A-levels and above	11.64	9.99	-
less than A-levels	8.37	5.61	-

Private sector workers are, on the whole, more mobile than public sector workers. Both high skill and low skill private sector workers have approximately a 0.1% better chance of changing jobs in a given month than their public sector counterparts. A closer look reveals that private sector workers experience much more frequent *within-sector* job changes than their public-sector counterparts. Mobility *between* employment sectors, however, is dominated by public sector employees moving to the private sector, cross-sector mobility in the other direction being a comparatively rare event. It is also striking to see that the separation rate into unemployment is only marginally smaller in the public sector than in the private sector. Finally, perhaps the most important conclusion to be drawn from Table 2 is that direct, job-to-job reallocation *between employment sectors* is substantial: given the transition rates in Table 2 and the various sectors' relative sizes given in Table 1, one can infer that about 20 percent of the employment inflow into the private sector comes from the public sector, and that about 30 percent of the private sector employment outflow goes into the public sector. High skill workers seem to have the best of both worlds, with higher rates of job movement and lower job destruction.

In addition, we have data on the distribution of firm sizes in the population of employed workers in the private sector. This data is provided by the Office of National Statistics[6] and relates to the

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[6]It is compiled from the Inter Departmental Business Register (IDBR) which contains information on VAT traders

Table 2: Job mobility within and between sectors

A-levels and above:	Private Sector	Public Sector	Unemployment
Private Sector	0.0153	0.0020	0.0044
Public Sector	0.0078	0.0077	0.0038
Unemployment	0.0865	0.0175	—
Less than A-levels:	Private Sector	Public Sector	Unemployment
Private Sector	0.0152	0.0010	0.0050
Public Sector	0.0101	0.0044	0.0041
Unemployment	0.0834	0.0080	—

Note: Transition rates are monthly. Rows do not add up to one. The two entries on the main diagonal are the fractions of workers *changing jobs* within the private and the public sector, respectively.

UK in 2010. The distribution of employers' sizes is reported in Table 3.

A caveat of this data is that it refers to employers' sizes in terms of all employees' skills combined, whereas our estimations are carried out on subsets of the data stratified by skills. Given our stylized modeling of the firm as a single-input constant-returns-to-scale production unit, we shall ignore this issue by assuming that either the distribution of firms' sizes among employees is the same across skill groups, or that the optimal number of hires h derived by a firm is shared in constant proportions between the different skill groups.

3.2 Estimation

Identification of the model's parameters $(b, a_g, \delta_p, \delta_g, \lambda_{up}, \lambda_{ug}, \lambda_{pp}, \lambda_{pg}, \lambda_{gp}, \lambda_{gg}, F_p, F_g, \Gamma, c(\cdot))$ comes from two data sources: observed transitions between labor market states and observed wage distributions. Data on the distribution of firms' sizes allow us to retrieve estimates of $(\Gamma, c(\cdot))$.

Observed sector-specific wage distributions are direct empirical counterparts to $G_p(\cdot)$ and $G_p(\cdot)$ in the model. While neither has a closed-form solution in the model, both can be simulated given parameter value. In order to map the wage distribution well, we take as moments to be matched 50 quantiles of the distributions, giving 100 moments in total: $\{w_{s,j}\}_{s=1,2,j=1..50}$.

Turning to transition moments, we match the eight transition rates reported in Table 2. Denoting these as $\pi_{ss'}$ where s is the state of origin and s' the state of destination, we thus add eight

Table 3: Employment in the UK private sector by firm size, 2010

Firm size	Employment (thousands)	Percent	Cumul
Total	22,514	100	
1	3,532	15.7	15.7
2	409	1.8	17.5
3 - 5	1,786	7.9	25.4
6 - 10	1,523	6.8	32.2
11 - 20	1,545	6.9	39.1
21 - 50	1,818	8.1	47.1
51 - 99	1,282	5.7	52.8
100 - 199	1,089	4.8	57.7
200 - 249	333	1.5	59.1
250 - 499	1,052	4.7	63.8
500 or more	8,147	36.2	100

Source: ONS.

moments to match: $(\pi_{up}, \pi_{ug}, \pi_{pu}, \pi_{pp}, \pi_{pg}, \pi_{gu}, \pi_{gp}, \pi_{gg})$. The theoretical counterparts of those monthly transition rates are given by the probabilities of a certain type of transition occurring within a one-month period. The theoretical counterparts of π_{pu}, π_{pp}, π_{pg}, π_{gu}, π_{gp} and π_{gg} all have similar expressions: $\pi_{ss'}^{\text{model}}$ is constructed by taking the probability that an exit from state $s = p$ or g, given wage w, occurs before one month has elapsed, multiplying it by the conditional probability of exiting toward s', given that an exit occurs and given initial wage w, then finally integrating out w using the relevant initial wage distribution, $dG_s(w)$. For example:

$$\pi_{pp}^{\text{model}} = \int_{R_{up}}^{+\infty} \frac{\lambda_{pp}\overline{F}_p(w)\left(1 - e^{-\left(\delta_p + \lambda_{pp}\overline{F}_p(w) + \lambda_{pg}\overline{F}_g(R_{pg}(w))\right)\times 1}\right)}{\delta_p + \lambda_{pp}\overline{F}_p(w) + \lambda_{pg}\overline{F}_g(R_{pg}(w))} dG_p(w),$$

where the "$\times 1$" term in the exponential is there as a reminder that π_{pp} is a monthly transition probability and that all the flow parameters (δ_p, λ_{pp}, etc.) are monthly. The theoretical counterparts of the transition rates from unemployment are simpler (as there is no wage to integrate out):

$$\pi_{up}^{\text{model}} = \frac{\lambda_{up}\left(1 - e^{-(\lambda_{up} + \lambda_{ug})\times 1}\right)}{\lambda_{up} + \lambda_{ug}},$$

and symmetrically for π_{ug}^{model}.

As for the estimation of the function $c(h)$, i.e. the cost of making h hires for a private sector firm,

we will be using the 10 cutoffs of the distribution of firms' sizes within private sector employment, denoted $\{H(\ell_i^c)\}_{i=1..10}$, where $H(\cdot)$ is the cumulative distribution function of firm sizes among private sector employees and the ℓ_i^c's are the size cutoffs displayed in the first column of Table 3. As will be discussed in the next section, the distribution of firm productivities in the population of firms, $\Gamma(y)$, will then be estimated by matching 50 points of the private sector wage *offer* distribution corresponding to the observed wage quantiles seen above, $\{F_p(w_{p,j})\}_{j=1..50}$.

3.3   Estimation Procedure

We first estimate the first twelve components of θ by matching the 108 moments described above, leaving $\Gamma(\cdot)$ out. The latter is backed out in a final step as the underlying private firm productivity distribution that rationalizes the estimates of $F_p(\cdot)$ and $F_p(\cdot)$ obtained in previous steps. We also make parametric assumptions about $F_p(\cdot)$ and $F_g(\cdot)$. First we assume that:

$$F_p(w) = \begin{cases} \frac{1-(w/R_{up})^{\alpha_p}}{1-(\overline{w}_p/R_{up})^{\alpha_p}} & \text{if } w \in [R_{up}, \overline{w}_p] \\ 0 & \text{if } w < R_{up} \\ 1 & \text{if } w > \overline{w}_p, \end{cases}$$

where $\overline{w}_p$ is set equal to the top percentile in the observed wage distribution. It then proves convenient to parameterize $F_g(\cdot)$ as:

$$F_g\left(R_{pg}\left[F_p^{-1}(x)\right]\right) = x^{\alpha_g}, \quad \text{for } x \in [0,1]. \tag{19}$$

Note that the latter parameterization carries the implicit assumption that the lower support of $F_g(\cdot)$ is precisely R_{ug}. This assumption, although not implausible, has no real theoretical justification as the public sector is not assumed to be profit maximizing and as such may offer wages that are all strictly greater than the workers' common reservation wage. Experimenting with richer specifications, allowing for the lower support of $F_g(\cdot)$ to be strictly above R_{ug}, led to the conclusion that those two numbers are indeed very close and that (19) is a valid approximation.

In order to match the moments described we implement a two-step algorithm. In a first step, we use the eight flow parameters $(\delta_p, \delta_g, \lambda_{up}, \lambda_{ug}, \lambda_{pp}, \lambda_{pg}, \lambda_{gp}, \lambda_{gg})$ to fit the eight transition rates derived from our model to those observed in the data, conditional on initial guesses about the offer

distributions $F_p(\cdot)$ and $F_g(\cdot)$. This first, just-identified step produces a perfect fit to observed transition rates. Then, in a second step, conditional on the transition rates obtained from the first step, we derive the offer distributions such that the distance between the vector of quantiles of the empirical and theoretical wage distributions $G_p(\cdot)$ and $G_g(\cdot)$ is minimized. The process is repeated until convergence. We find that this iterative two-step protocol produces a better overall fit than one-step procedures.

Now turning to the estimation of the last two components of θ, namely $c(h)$ and $\Gamma(y)$, we use the fact the larger firms pay higher wages, i.e. ℓ is increasing in w, which is consistent with our model (see appendix). This and data on $H(\ell_i^c)$ allows us to infer the wage rates w_i^c paid at each cutoff size ℓ_i^c:

$$G_p(w_i^c) = H(\ell_i^c) \quad \text{for } i = 1..10, \tag{20}$$

where $\ell_i^c = \ell(w_i^c, h_i^c) = \ell(w^\star(y_i^c), h^\star(y_i^c))$. Now, with the pair (ℓ_i^c, w_i^c) at the 10 cutoff sizes, we are able to infer both h_i^c and $c'(h_i^c)$ at these 10 points thanks to the following (see equations (11) and (14)):

$$
\begin{aligned}
h_i^c &= \ell_i^c \Delta(w_i^c) \\
c'(h_i^c) &= \frac{-1}{\Delta'(w_i^c)}
\end{aligned}
$$

We thereby obtain a non-parametric estimate of the shape of $c'(\cdot)$ over a set of 10 points, from which we extrapolate the derivative of the cost function over the whole range of wages[7].

All that is left to estimate now is the distribution of productivities in the population of firms, $\Gamma(y)$. Productivity levels are derived from the first-order condition (13), which gives us a relationship $\tilde{y}(w)$, where $\tilde{y}$ is such that $w^\star(\tilde{y}) = w$. The number of hires $h^\star(y)$ is estimated by inverting the cost of hire function, equation (14), thus giving us an $h^\star(y)$. Manipulation of the expression for the wage offer distribution, equation (16) gives us an expression for $\Gamma(y(w))$, this performed in the appendices.

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[7]Note that we can retrieve the cost function itself by assuming that the cost of making zero hires is zero and by integrating c'.

$$\Gamma\left(y\left(w\right)\right) = M \int_0^{F_p(w)} \frac{\tilde{\alpha}\left(x\right)}{\tilde{h}\left(x\right)} dx \tag{21}$$

Thus we obtain the $\Gamma(\cdot)$ distribution that matches 50 points of the F_p distribution previously estimated.

4 Results

4.1 Labor Market Transitions

Parameter estimates of the model are given in Table 4, of which the top panel contains all transition parameters. The unit of time associated with the transition/offer arrival rates is a month. Again, given our estimates of the rest of the parameter vector, those transition rate values produce a perfect fit to the observed monthly transition rates reported in Table 2.

A striking feature of our parameter estimates is the large on-the-job offer arrival rates for workers in both sectors. Comparison of our estimates of λ_{pp}, λ_{pg}, λ_{gp}, and λ_{gg} with the corresponding monthly transition rates π_{pp}, π_{pg}, π_{gp}, and π_{gg} (see Table 2) suggests that employed workers only accept between 2.5 and 3.5 percent of the offers they receive. Moreover, employed workers, regardless of sector, receive far more frequent offers from both sector than unemployed workers. Those results contrast sharply with standard findings from simple, one-sector wage-posting model. However, the same pattern arises in estimates obtained by Meghir et al. (2010) in a different two-sector wage posting model, with a formal and an informal sector estimated on Brazilian data.

Given in the parenthesis are the 95% confidence intervals for the parameter estimates. These are obtained by resampling the data, allowing for repetition and running the estimation protocol outlined previously on repeated redraws of the data. Transition rates will be non-negative by construction across all redraws, thus so will the transitional parameters and their distributions will be non-symmetric, as can be seen in Table 4. We therefore find displaying confidence intervals more informative than standard errors.

Table 4: Parameter Estimates

| A-Levels and above			
δ_p	λ_{up}	λ_{pp}	λ_{pg}
0.0044	0.0913	0.7160	0.1157
(0.0039,0.0054)	(0.0768,0.1063)	(0.3327,1.5937)	(0.0331,0.2996)
δ_g	λ_{ug}	λ_{gp}	λ_{gg}
0.0039	0.0185	0.3661	0.4369
(0.0031,0.0060)	(0.0126,0.0250)	(0.1427,0.7870)	(0.1129,1.0956)
α_p	α_g	b	a
2.9958	0.8208	11.16	-1.42
(2.7586,3.2298)	(0.2815,3.4827)	(9.7215,11.8487)	$(-2.4755,0.1276)$
Less than A-Levels			
δ_p	λ_{up}	λ_{pp}	λ_{pg}
0.0050	0.0875	0.3523	0.0294
(0.0044,0.0063)	(0.0726,0.1026)	(0.2057,0.6379)	(0.0077,0.0524)
δ_g	λ_{ug}	λ_{gp}	λ_{gg}
0.0042	0.0084	0.2210	0.1156
(0.0026,0.0078)	(0.0049,0.0123)	(0.0898,0.4832)	(0.0356,0.2735)
α_p	α_g	b	a
3.5016	0.7972	7.45	-1.09
(3.2544,4.1731)	(0.6487,3.3715)	(6.4323,8.2692)	$(-2.4745,-0.2823)$

4.2 Wages and Worker Values

Table 4 also reports estimates of the wage offer distribution parameters, α_p and α_g, and an estimate of the flow value of unemployment, b, at about £11/hr for high skill workers and £7.50/hr for low skill. The latter implies values of the unemployed workers' reservation wages of R_{up} =£2.29/hr (R_{up} =£2.74/hr) and R_{ug} =£4.75/hr (R_{ug} =£4.85/hr) for high skilled workers (low skilled workers), respectively. The fact that R_{up} and R_{ug} are both lower than b, which is unusual in empirical wage posting models, is a consequence of the relative values of offer arrival rates on- and off-the job: unemployed workers are prepared to give up a substantial amount in terms of income flow to benefit from the more efficient on-the-job search technology. It is also the higher job arrival rate coupled with lower job destruction that means high skilled workers have lower reservation wages from unemployment. There is a small public sector *stigma*, but it is small relative to wages and is not statistically significantly different from zero for the higher skilled workers.

We now turn to an analysis of wage distributions. Panels (a) in Figure 1 shows the model's fit to observed cross-section (log-) wage distributions in both sectors of employment. The fit is
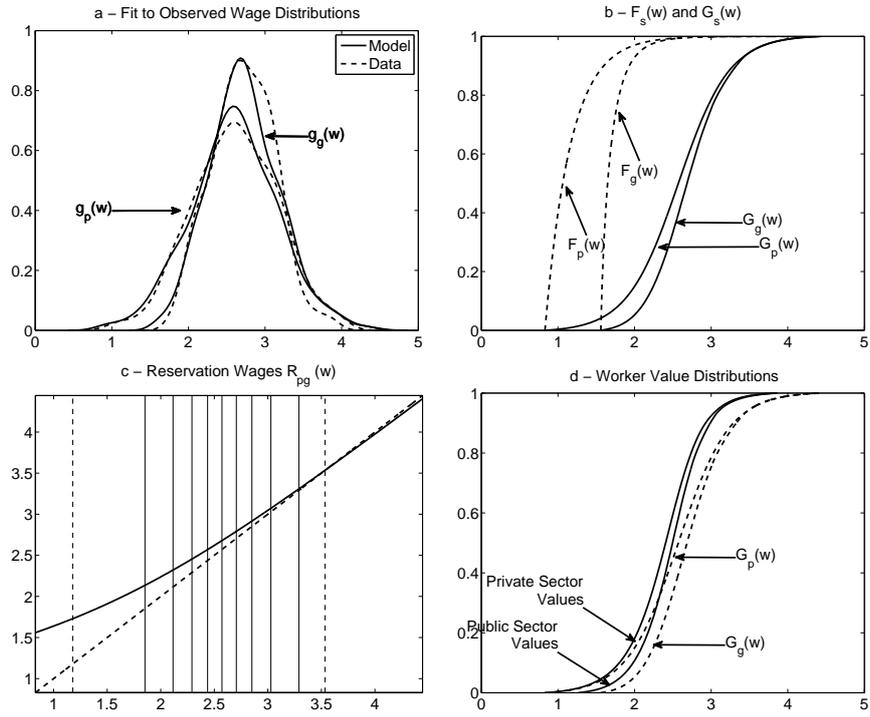
A-Levels and above
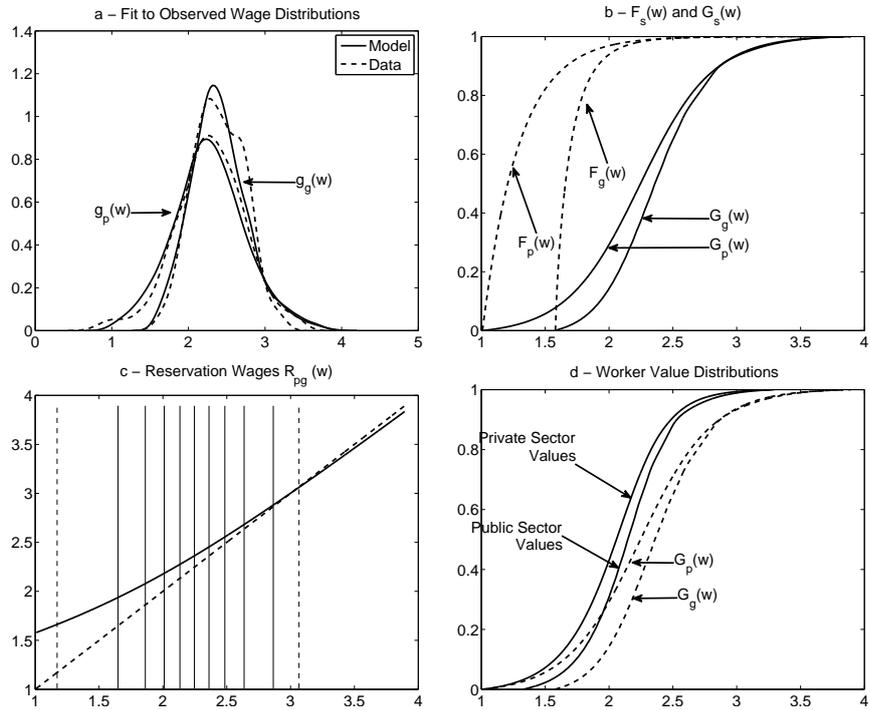
Less than A-levels

Figure 1: Wage distributions and reservation wages

reasonably good in both sectors, although the model has some difficulty fitting the high quantiles of the public-sector distribution. In passing, we note that, as is well documented elsewhere in the literature, the public-sector wage distribution dominates the private-sector one except in the top two deciles. There is also markedly less wage dispersion in the public sector.

Panels (b) in Figure 1 next shows estimated log-wage offer distributions in both sectors, $F_p(\cdot)$ and $F_g(\cdot)$, together with the distributions of accepted log-wage offers, $G_p(\cdot)$ and $G_g(\cdot)$. Both offer distributions are fairly concentrated, much more so than the corresponding accepted offer distributions. Indeed the large estimated offer arrival rates imply a large extent of stochastic dominance of $G_s(\cdot)$ over $F_s(\cdot)$ for $s = p$ or g. We also see that the public-sector offer distribution strongly dominates its private-sector equivalent. However part of that dominance is "un-done" by later reallocation within and between sectors: the dominance of $G_g(\cdot)$ over $G_p(\cdot)$ is less marked than the one of $F_g(\cdot)$ over $F_p(\cdot)$. The main driver here is the rate λ_{pp} at which private sector workers receive offers of alternative jobs in the private sector, which is very high compared to it public sector counterpart λ_{gg}, implying much quicker upward wage mobility in the private than in the public sector. As a consequence, the distribution of private sector wages $G_p(\cdot)$ dominates the private sector offer distribution $F_p(\cdot)$ by much more than $G_g(\cdot)$ dominates $F_g(\cdot)$.

Panels (c) in Figure 1 then plots $R_{pg}(w)$, the reservation wage of private-sector employees presented with offers from the public sector. The dashed line on that graph is the main diagonal and the vertical lines materialize the deciles of the private-sector wage distribution, $G_p(\cdot)$. It appears on this plot that, over most of the support of $G_p(\cdot)$ (up to about the 95th percentile), $R_{pg}(w) > w$, i.e. private-sector employees will only accept to go into the public sector with a wage increase. The likely reason is again that upward wage mobility is quicker in the private sector, mainly because of the high value of the private-sector offer arrival rate λ_{pp}.

Finally, our model allows us to examine the public-private sector pay gap. While this pay gap is conventionally assessed in terms of wages, in our model workers care not only about their wages but also about future wages, which depend on transition rates and expected future wage progression patterns that differ between sectors. Following Postel-Vinay and Turon (2007), we thus assess the

public-private pay gap in terms of lifetime values of employment, as well as raw wages. Specifically, panels (d) in Figure 1 takes up a plot of the private- and public-sector wage distributions $G_p(\cdot)$ and $G_g(\cdot)$, together with corresponding plots of the distributions of worker "permanent income" in both sectors, where we define permanent income as the annuitized worker value $rW_p(w)$ and $rW_g(w)$ as defined by equation (4). Again, the x-axis on panels (d) is on a logarithmic scale. We draw two conclusions from this graph. First, assessing the public pay gap based on wages only may lead one to overstate the public sector premium: the public- and private-sector distributions of worker values are much closer together than the corresponding wage distributions. In other words, differences in patterns of wage mobility (due to differences in offer arrival rates) between sectors rub off a large part of the public-sector wage premium. Second, even when comparing lifetime values, a public premium persists in the lower quantiles of the value distribution in the sense that the distribution of public-sector values dominates its private-sector equivalent in the bottom quantiles, consistently with the documented fact that low-wage workers tend to queue for public sector jobs much more than high wage workers. This dominance is particularly pronounced for low skilled workers. Both of those conclusions are consistent with the findings of Postel-Vinay and Turon (2007), who estimate a partial equilibrium, descriptive model of wages on job mobility, also on BHPS data, but with a much richer representation of worker heterogeneity and wage dynamics.

5 Counterfactual Policy Analysis

Using the estimated parameters of the structural model we simulate the effects of various changes in public sector wage and employment policy. While the model allows the simulation of many possible policy changes, from a topical perspective, an assessment, by simulations, of the various public sector austerity measures being enacted across Europe seems to be a sensible subject to pursue. As a benchmark we will look at the effect of the UK government's attempt to cut their deficit through increasing public sector layoffs, the goal of which was to reduce public sector employment by almost half a million, approximately 8%. To determine whether this is sensible policy making we will compare this with policies undertaken by other European governments. Spain is attempting to

reduce public spending through reducing public sector pay and Italy by reducing new public sector hires. In the context of our model, changes to public sector hiring is modeled as changes to λ_{sg} and wages to changes in $F_g(w)$.

5.1 Simulation Protocol

Previously the wage offer distribution of the public sector ($F_g(w)$) was parameterized as a function of the wage offer distribution of the private sector ($F_p(w)$), equation (19). As $F_p(w)$ is an endogenous object it will change with changes to public policy, we therefore need to fix $F_g(w)$ ex-ante. Taking the point estimates from estimation we fit the distribution derived from the transform of equation (19) and the distribution in equation (22). We include the shifting parameter $\tilde{\alpha}$ so as to match the distribution more closely as the lower bound R_{ug} also affects the curvature of the distribution.

$$
F_g(w) = \begin{cases} \frac{1-(w+\tilde{\alpha}/R_{ug})^{\tilde{\alpha}_g}}{1-(\overline{w}_g/R_{ug})^{\tilde{\alpha}_g}} & \text{if } w \in [R_{ug}, \overline{w}_p] \\ 0 & \text{if } w < R_{ug} \\ 1 & \text{if } w > \overline{w}_g, \end{cases} \tag{22}
$$

We implement policy changes as changes to the job offer arrival rates of public sector jobs, public sector job destruction and the wage offer distribution. Simulating the new equilibrium is performed using an iterative procedure:

1. When first implementing these changes we take a relatively broad set of initial values of the wage offer and offer arrival rate workers receive from the private sector. Specifically, we take an interval around the estimated values of the Pareto index (α_p) of the $F_p(\cdot)$ distribution and similarly for private sector job offer arrival rate (λ_p), such that $\alpha_p^0 \in [\alpha_p - 0.5, \alpha_p + 0.5]$ and $\lambda_p^0 \in [\lambda_{up} - 0.05, \lambda_{up} + 0.05]$. We thus construct an $(n \times n)$ grid, where n is the number of discrete evenly spaced initial values of α_p and λ_p we consider.

2. First, the worker side is re-evaluated as before, with new parameters equations (6) through (9) are solved as before, giving us new values of N_u, N_p, N_g, $G_p(w)$, $G_g(w)$, R_{up}, R_{ug} and $R_{pg}(w)$.

3. Turning to the firm side: the cost of hires and firm productivity distributions are treated

as primitives of the model. Firms maximize profit according to their first order conditions, equations (13) and (14). Conditional on α_p^0, λ_p^0 and the parameters calculated in step 2 the new optimal search policy $h^\star(y)$ is derived. Then our update $F_p(\cdot)$ is obtained from equation (16) and $\lambda_p^{(n,n)}$ from equations (15) and (18). Then fitting a Pareto distribution through the updated $F_p(\cdot)$ one obtains $\alpha_p^{(n,n)}$.

4. Once a second grid of the updated pair (α_p, λ_p) has been constructed. The fixed point is the element of the grid for which the distance between the updated parameter pair and the initial guess is closest to zero.

5.2   Counterfactual Policies

Using the estimated parameters of the structural model we simulate the effects of various changes in public sector wage and employment policy. It seems sensible to use our model to assess the austerity measures being implemented across Europe. As a benchmark we will look at the effect of the UK government's attempt to cut their deficit through increasing public sector layoffs. The goal of which was to reduce public sector employment by 490,000 over four years, approximately 8% of the public sector. To determine whether increase layoffs is the most efficient policy option we will compare it with policies undertaken by other European governments during the "*age of austerity*"[8].

Clearly even within country there is a great variety in policies undertaken. In this section we take a simplistic view that each country attempts to reduce its public sector wage bill with one-dimensional policy. Alternatives to Cameron's cuts are Mario Monti's reduction in public sector recruitment in Italy and the progressive wage policies implemented by Portugal's center-right coalition. The policies will inevitably have many effects on the wage bill, the employment level and inequality and one metric to determine optimality doesn't seem feasible. So to determine success we assess the associated costs in employment, inequality and wage level with approximately the same cut to public spending.

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[8]The phrase was coined by Prime Minister David Cameron at the Conservative party forum in Cheltenham, 26th April 2009

Table 5: Counterfactual Policy Effects

	No Change		UK Policy		Italian Policy		Portuguese Policy	
	high skill	low skill	high skill	low skill	high skill	low skill	high skill	low skill
Employment								
Unemployed	0.0378	0.0492	0.0377	0.0534	0.0384	0.0518	-	-
Private Sector	0.7626	0.8576	0.771	0.8558	0.7728	0.8596	-	-
Public Sector	0.1995	0.0932	0.1913	0.0908	0.1889	0.0887	-	-
Mean of Wages								
Private Sector	15.4727	10.7195	15.4227	10.7755	15.4537	10.7512	-	-
Public Sector	16.9474	11.8422	16.946	11.6118	16.9587	11.7273	-	-
St. Dev of Wages								
Private Sector	11.3198	7.031	11.5629	7.3824	11.1987	7.1001	-	-
Public Sector	9.2827	5.3239	9.5209	5.0759	9.4344	5.0781	-	-
Discounted Present Value								
Unemployment	2.2939	2.742	2.2903	2.7295	2.2807	2.731	-	-
Private Sector	24.2292	16.6117	24.1571	16.6734	24.162	16.6331	-	-
Public Sector	31.0616	20.0767	30.4241	19.4266	30.8906	19.8414	-	-
Savings (%)	0	0	4.1132	4.4405	5.2858	5.7931	-	-

The policies we implement are as follows: the UK policy is a 5% increase in the public sector job destruction rate; the Italian policy is a 5% increase in all public sector offer arrival rates and the Portuguese is a progressive reduction in offered public sector wages culminating in a 5% lower wage for the highest paid workers. The new policies are denoted by a $\star$ in equation (23).

$$\text{UK: } \delta_g^\star = 1.05 \times \delta_g$$

$$\text{Italy: } \lambda_{sg}^\star = 0.95 \times \lambda_{sg} \text{ where } s \in \{u, p, g\}$$

$$\text{Portugal: } w_g^\star = \frac{0.95 \times \overline{w}_g - \underline{w}_g}{\overline{w}_g - \underline{w}_g} + \underline{w}_g \tag{23}$$

All three of the policies we have examined have minimal impact on the overall employment level. The small decreases in the size of the public sector are largely offset by increases in the private sector. In the case of increased public sector layoffs (UK policy) for high skilled males the private sector expands by more than public sector contracts, reducing the overall unemployment rate. As one may expect increasing fires (UK) and reducing new recruitment (Italian) have fairly

similar effects with the former putting a larger burden of austerity on the low skilled workers than the latter. Both also have negligible impact on the mean wages in either sector but have some effect on the second moments of the wage distributions.

There is enormous scope to examine changes to the menu of wages offered in the public sector. We have chosen a progressive wage decrease as this has been used most widely in the context of austerity, notably by the Portuguese, Irish and Spanish governments.

Public sector wage policy can be a very flexible policy tool which can be examined in the context of our model. Not only in determining who bears the cost of austerity measures but in other contexts as well. Eurozone countries no longer have autonomy over monetary policy. One way of depreciating themselves could be by reducing wages in the public sector, which will have a knock on effect in the private sector. In order to exactly determine sensible policy one would need a model akin to this. We feel this presents a very exciting avenue for future policy studies.

6   Concluding Remarks

COMING SOON...

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A Theory: Intermediate Derivations

A.1 Derivation of the Reservation Wage, Equation (6)

The value function for a private sector worker earning a wage w, is given in equation (4). Assuming differentiability:

$$W'_p(w) = \left[r + \delta_p + \lambda_{pp}\overline{F}_p(w) + \lambda_{pg}\overline{F}_g(R_{pg}(w))\right]^{-1} \tag{24}$$

This also gives $W'_p(w)$ by analogy. Integrating by parts in (4) yields:

$$(r + \delta_p)W_p(w) = w + \delta_p U + \lambda_{pp}\int_w^{+\infty} W'_p(x)\overline{F}_p(x)\,dx + \lambda_{pg}\int_{R_{pg}(w)}^{+\infty} W'_g(x)\overline{F}_g(x)\,dx \tag{25}$$

Plugging the various value functions into the definition of $R_{pg}(w)$ given in the paper, one obtains the following, fairly complicated expression:

$$R_{pg}(w) = -a_g + \frac{r+\delta_g}{r+\delta_p}w + \left\{\frac{r+\delta_g}{r+\delta_p}\delta_p - \delta_g\right\}U + \left\{\frac{r+\delta_g}{r+\delta_p}\lambda_{pp} - \lambda_{gp}\right\}\int_w^{+\infty} W'_p(x)\overline{F}_p(x)\,dx$$
$$+ \left\{\frac{r+\delta_g}{r+\delta_p}\lambda_{pg} - \lambda_{gg}\right\}\int_{R_{pg}(w)}^{+\infty} W'_g(x)\overline{F}_g(x)\,dx$$

Differentiating yields (6).

A.2 Derivation of the Private-Sector Wage Distribution, Equation (9)

Equation (9) would be a simple ODE if it was not for the term featuring $G_g(R_{pg}(w))$. We now show how to express that term as a function of w and $G_p(w)$. Writing the flow-balance equation for the public sector yields:

$$\left\{\delta_g + \lambda_{gg}\overline{F}_g(w)\right\}N_g G_g(w) + N_g\lambda_{gp}\int_{R_{ug}}^w \overline{F}_p(R_{gp}(x))\,dG_g(x)$$
$$- N_p\lambda_{pg}\int_{R_{up}}^{R_{gp}(w)} [F_g(w) - F_g(R_{pg}(x))]\,dG_p(x) = N_u\lambda_{ug}[F_g(w) - F_g(R_{ug})].$$

Now applying the latter equation at $R_{pg}(w)$ (instead of w), we get:

$$\left\{\delta_g + \lambda_{gg}\overline{F}_g(w)\right\}N_g G_g(R_{pg}(w)) + N_g\lambda_{gp}\int_{R_{ug}}^{R_{pg}(w)} \overline{F}_p(R_{gp}(x))\,dG_g(x)$$
$$- N_p\lambda_{pg}\int_{R_{up}}^w [F_g(R_{pg}(w)) - F_g(R_{pg}(x))]\,dG_p(x) = N_u\lambda_{ug}[F_g(R_{pg}(w)) - F_g(R_{ug})]. \tag{26}$$

Adding (26) to (8):

$$N_p G_p(w) \{\delta_p + \lambda_{pp} \overline{F}_p(w) + \lambda_{pg} \overline{F}_g(R_{pg}(w))\}$$

$$+ N_g G_g(R_{pg}(w)) \{\delta_g + \lambda_{gp} \overline{F}_p(w) + \lambda_{gg} \overline{F}_g(R_{pg}(w))\}$$

$$= N_u \lambda_{ug} [F_g(R_{pg}(w)) - F_g(R_{ug})] + N_u \lambda_{up} [F_p(w) - F_p(R_{up})], \quad (27)$$

which can be solved for $N_g G_g(R_{pg}(w))$. Plugging the solution into (9), we obtain an ODE defining $G_p(w)$. Note that by considering $w \to +\infty$ in the latter equation, one obtains (7).

A.3 Productivity Distribution

Using a change of variable and assuming the that $w^\star(y_{min}) = R_{up}$, equation (16) is equivalent to:

$$F_p(w) = \frac{1}{M} \int_{R_{up}}^w \frac{h(z)}{\alpha(z)} \gamma(y(z)) y'(z) \, dz \tag{28}$$

$$f_p(w) = \frac{1}{M} \frac{h(w)}{\alpha(w)} \gamma(y(w)) y'(w)$$

$$\frac{d}{dw} \{\Gamma(y(w))\} = \frac{M f_p(w) \alpha(w)}{h(w)}$$

$$\Gamma(y(w)) = M \int_{R_{up}}^w \frac{\alpha(x)}{h(x)} dF_p(x)$$

$$\Gamma(y(w)) = M \int_0^{F_p(w)} \frac{\tilde{\alpha}(x)}{\tilde{h}(x)} dx$$

Where, $\alpha(x) = \tilde{\alpha}(F_p(x))$, $h(x) = \tilde{h}(F_p(x))$ and $h(w) = c'^{-1}\left(\frac{y-w}{\Delta(w)}\right)$