The Source of Wage Progression Over the Life Cycle in France: Wage Game, Human Capital Accumulation and Age-dependent Contribution of the Unemployment Insurance System

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Abstract

The search effect is traditionally considered as a major source of wage progression over the life cycle. In France, high wage progression over the life cycle and low mobility rate coexist. To solve this puzzle, in this paper, we investigate the possible sources of wage progression in France. To do so, we introduce a finite horizon into the framework developed in (Mortensen, 1998), with frictions, on the job search, endogenous matches’ productivity resulting from a specific human capital investment of firms and in which wages are posted. By augmenting this framework with human capital accumulation, and progressive unemployment benefits, we achieve to decompose the French wage progression into three channels: the wage game, the human capital accumulation and the institutional channel. Thanks to calibration on French data, we show that the puzzle of French wage progression can be explained by the wage-indexed unemployment insurance system. Over their working life, workers accumulate rights to the unemployment insurance system. The workers’ labor opportunity costs raise over their life cycle and so their market power. In the same time, their presence reduces the number of job openings and therefore reduces workers’ mobility. We also show that the institutional channel contracts consequently the wage distribution of workers of the same age: French institutions therefore raise the wage inequality between the older workers and the young workers yet decrease the wage inequalities within the age classes.

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†I thank ... Any errors and omissions are ours.
1 Introduction

The theoretical frameworks developed in (Burdett and Mortensen, 1998), (Mortensen, 1998) and (Postel-Vinay and Robin, 2002) have become today major references to explain wage distribution. In these theoretical works, the key mechanism of workers’ wage increase is the on the job search and the job to job transitions it yields. French labor market is known for its particularly low job to job mobility rate: in 2002, when a U.S. worker had on average 21% of chance to move from one job to another in one year, his French counterpart had only 9%. Yet, surprisingly enough, average wage progression over the life cycle is slightly higher in France than in the U.S.: around 1.34% per year in France for 1.25% in the U.S.. Besides, French wage progression occurs all over the life cycle as figure 1 shows it, whereas workers mobility tends to decreases all over the life cycle: the rate of mobility of workers aged between 46 to 58 years old is only 6%. This puzzle justified the investigation of the source of wage progression in France.

Figure 1: Wage distribution for French salaried men by age class (First 95%), expressed in French minimum wage

French labor market differs from the U.S.’s by the presence of strong institutions like the wage-indexed unemployment insurance system. The bismarckian component of this system allows workers to accumulate rights to the unemployment insurance system all over their working life. The French institutional environment therefore allows workers to progressively improve their outside options and could account for a part of
observed wage progression in France without high workers’ mobility. In this paper, we build a model which allows us to decompose the evolution of the French wage distribution over the workers’ life cycle into two mechanisms: the shift in workers’ labor market power and the return of job to job mobility over their life cycle, and into three channels: the wage game channel, which is the wage evolution at equal ex ante workers’ productivity, the human capital channel, which represents the increase in wage induced by the accumulation of human capital, and the institutional channel, which account for the unemployment insurance system effect.

To do so, we augment the wage posting model of (Mortensen, 1998), which is today the reference to explain the observed wage distribution, with a stylized life cycle of three age classes. We assume a segmented labor market, i.e. the search is directed on the three age classes\(^1\). As in (Mortensen, 1998), workers can search on the job and firms can invest on specific human capital and in search (by opening or not vacancies). Yet given the life-cycle dimension, we also assume workers can accumulate or lose human capital over their working life (Becker, 1964). Eventually, unemployed workers receive benefits depending on their previous wage as in (Chéron and Langot, 2010). Quantitative evaluations of the model are made on 2002 French data (Enquete Emploi).

Since the seminal (Burdett and Mortensen, 1998) paper, it is usual to use a wage posting game in a labor market with frictions to explain the observed wage distribution (see (Mortensen, 2003)). In this model, firms compete to capture a fraction of the workforce: they have the monopsony power to post wages. Worker can be unemployed or employed but search in each state a better wage offer. Since workers can change jobs to improve their wages, the power of firms is reduced, exerting upward pressure on wages. Firms also compete to retain their employed workers and avoid extra search costs. At equilibrium, the (Diamond, 1971) paradox is solved: there exists a wage dispersion in the search model explained by the job to job mobilities. Besides, (Mortensen, 1998) shows that by augmenting this framework with the endogenous productivity of firms, the model could provide with a realistic wage distribution with only a very mild restriction on the shape of the production function\(^2\). (Chéron, Hairault, and Langot, 2008) show that this search model with endogenous productivity is not rejected by the data. (Mortensen, 1998) also makes the synthesis between this branch of search models studying wage distribution and the other branch studying labor market’s flows by endogenizing the number of firms in the market in (Burdett and Mortensen, 1998)

\(^1\)This assumption is supported by the fact that firm may require or not a minimum of experience when it posts a vacancy. Thus, even if age discrimination is prohibited, the young can be excluded from the labor market of the older (they have not the minimal experience), whereas the older are physically inapt to insure a task ask to a young worker.

\(^2\)Only a decreasing return of human capital allows to reproduce the hump shape of the wage distribution
by introducing a free entry condition. In the wage game of (Mortensen, 1998), ex ante homogenous firms choose whether to invest in search cost (constant) and if they do, then decide on both a wage and an amount of specific human capital to invest in the match each time they hire a worker. This last investment can be associated to a vocational training of the employee at the beginning of the match. This specific human capital is costly and increases the match’s productivity during all its tenure. In this wage game, the market power of workers becomes the capacity of workers to force firms to raise their wage offers and therefore to create high quality jobs. This assumption reinforces the investment dimension of wage in this wage game: a high retention induced by a wage at the top of the distribution allows to both avoid extra search costs induced by poaching, and amortize a large specific human capital investment.

In this paper, we introduce an age-heterogeneity among the workers’ population in this wage game. The wage game channel consists therefore in the evolution of this wage game over three workers’ life periods. There exists a large difference between the beginning of the life-cycle of an agent and his end. At the beginning of the life cycle, the agent enters in the labor market as an unemployed worker. In addition, the youngest agents have not had the time to largely improve their careers. At the opposite, at the end of the life cycle, a large majority of workers are integrated in the firms and their experiences have given them the opportunities to find the better wage offers. Given workers’ mobilities, the time spent on the labor market, and therefore the workers’ age, necessarily affects the wage game described in (Mortensen, 1998). Yet, this backward dynamic must be combined with the forward looking behaviors of agents: introducing age supposes to introduce a retirement age. Young workers have a long horizon, whereas older workers have a short one. Given the investment dimension of (Mortensen, 1998)’s wage game, assuming a finite horizon in this model is not neutral. It can for instance account for the lower seniors’ mobility rate observed in the data \(^3\) by decreasing firms’ profit on the seniors’ market and discouraging firms to enter older workers’ market. From both backward and forward aspects, the wage game depends on the workers’ age.

Yet, wage progression and low seniors’ mobility rate can also be explained by the evolution of human capital which can be accumulated over most of the workers’ life (Becker, 1964) and become eventually obsolete at the end of it (Aubert, Caroli, and Roger, 2004). This alternative explanation of the main changes in labor market outcomes over the life cycle is also taken into account in our model by introducing the human capital channel. We choose to have no a priori on the trend of the general

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3This model does not take into account the search effort of workers, considering it would go in the same way as our modelization and would only reinforce our results ((Hairault, Langot, and Sopraseuth, 2010) and (Chéron, Hairault, and Langot, 2011))
human capital evolution since theoretically, aging can allow learning by doing, and also general human capital depreciation. The parameters representing the path of the general human capital over the three life period are estimated by the model. In our model, the level of general human capital affects the workers’ productivity by two aspects: first, through the workers’ productivity at their workplace, and second through the cost of the firms’ specific human capital investment. The first component represents the workers specific part of the productivity. The second component highlights the complementarity between the general human capital of the worker and the specific human capital of the firm: the more skilled is the worker the more firms are induced to train him.

Eventually, to account for the institutional channel, we augment this framework with progressive unemployment benefits as modelized in (Chéron and Langot, 2010).

The contribution of this paper is threefold. First it has an original contribution in explaining workers’ wage progression by using institutions like wage-indexed unemployment benefits. Up to now, it seems that no other work studies this aspect of unemployment insurance. Second, it brings a new light to the debate on the share of human capital accumulation to wage progression. Indeed, the developed framework allows us to decompose the workers’ wage progression into three channels: the evolution of the wage game (the pure effect of age on the wage distribution when workers are assumed ex ante evenly productive), the general human capital evolution and the age-dependent effect of the unemployment insurance system, while taking into account the finite horizon of workers. The working paper of (Bagger, Fontaine, Postel-Vinay, and Robin, 2012) explains the wage progression via job to job mobilities and human capital accumulation in the theoretical framework developed by (Postel-Vinay and Robin, 2002), yet in a infinite horizon. Their decomposition of individual wage growth reveals that human capital accumulation is the most important source of wage growth in early phases of workers’ careers, and search-induced wage growth in the second part of the life cycle. (Menzio, Telyukova, and Visschers, 2012) deal with this link in finite horizon and distinguish the same two channels of wage evolution. Their results show on the contrary that wage progression can only occur at the end of the life cycle thanks to human capital accumulation. Yet, these authors use a framework very different from the (Burdett and Mortensen, 1998), in which the effect of frictions is very weak, we are actually very close to a competitive model, the employment conditions of workers at one date does not affects the choice of agents of the next date and where there is no specific human capital investment. Eventually, this work has a theoretical contribution. Indeed, introducing a social finite horizon modifies deeply the wage posting game described in (Mortensen, 1998): the matches’ surplus and therefore the wage game become non stationary. New trade-offs therefore emerge: on the wage
strategy of the firm, on the productivity of matches and on the firms’ search intensity. All these firms’ decisions are affected by both backward and forward forces.

The model developed in this paper is calibrated on French data and fits rather well the data of the aggregated wage distribution and its evolution with workers’ age. We use the calibrated model to decompose the wage progression with age into the tree channels described earlier. The evolution of the wage game over the life cycle represents the pure effect of age on the wage distribution. According to our findings, the wage game contributes very lightly to the wage growth in the first half of the working life, and contributes negatively to this growth in the second half of it. When workers are ex ante evenly productive, the forward dynamics dominates the backward one. The firms’ expected match surplus decreases with workers’ age since nothing changes but the progressive decrease of the expected job duration. On the seniors’ market, firms are therefore fewer, compete less to retain workers who can leave exogenously soon, and invest less in specific human capital since it is unlikely that a high investment pays for itself given their short horizon. The human capital channel contributes positively to wage progression all over the workers’ life cycle: learning by doing dominates human capital depreciation. Human capital accumulation is particularly crucial in the second part of the working life since it allows to compensate the unfavorable condition of seniors induced by their short horizon. As in (Postel-Vinay and Robin, 2002), we find that the workers’ productivity accounts for a great part of the wage growth in the first part of the life cycle, yet in our estimation, its contribution to the wage growth in the second part of the life cycle is, although lower, very consequent since without it, wage would decrease. This divergence of results is easily explicable since they do not take into account the finite horizon of workers. By raising the outside options of workers in an increasing way with age, the unemployment insurance system contributes consequently to wage progression. Besides, this unemployment insurance system-induced wage progression occurs inhomogenously over the life cycle: on the 1.48% of average wage increase in the first part of the working life they contribute by 0.10 points of percentage and on the 1% of average wage increase in the second part of the working life by 0.25 points of percentage. Eventually seniors are those who benefit the most from the effect of unemployment benefits on wages. Yet, the unemployment insurance system also accounts for both a decrease of the search activity and of its outcome. The unemployment insurance system-induced wage progression in France therefore partly substitute the search-induced wage progression. Besides, in France, if the unemployment insurance system raises wage inequality between the older workers, it decreases wage inequalities within age classes. In the second section, we present the assumptions of the model. The equilibrium definition is given by the third section. Section four is dedicated to the description of the data, the model’s calibration.
and validation on French data. Section 5 gives the results of the wage progression decomposition into the wage game channel, the human capital channel and the institutional channel. Section 6 concludes.

2 Model assumptions

2.1 Labor market setup and main notations

We introduce life-cycle in the (Burdett and Mortensen, 1998) theoretical framework. The life-cycle is cut in three working life periods, namely the young, the adults, and the seniors. We choose the three age classes segmentation in order to stay close to the main characteristics of the life-cycle data: the integration to the labor market, the maturity, and the seniority. All variables dependent on the workers’ age class are indexed by \( i \), which can take the value \( i = y \) for the young, \( i = a \) for adults and \( i = s \) for seniors. Age classes changes are stochastic. The workers change age class with the probability \( p \). As this probability is the same between each age class, in steady state the mass of workers noted \( m \) of each age class is the same. As in (Burdett and Mortensen, 1998), we only consider steady state and assume time is continuous. Between these three age periods, we allow workers to accumulate human capital. This accumulation affects the training cost paid by the firms \( \beta_i \) and their productivity at the workplace \( y_i \). The trend of \( \beta_i \) and \( y_i \) over the workers’ life is a priori unknown and is to be estimated by the model.

As in (Burdett and Mortensen, 1998), wages are posted by firms, and workers take the job or leave it. There is no negotiation over the wage. Firms set wages in order to maximize their profits knowing that they cannot observe nor the status or the reservation wage of workers: information is not perfect. Firms direct their search on workers’ age class. The employer can observe the workers’ age class and discriminate them on this criterion. When a firm enters one of the three markets, production generated by employing a worker from the two other markets is null. Therefore, workers do not cheat. When workers change age class, the contract is not broken unless the workers’ value of keeping the contract obtained in the previous age period becomes lower than the value of being unemployed in his current age period. Firms which target the youth can therefore be exposed to employ senior workers eventually. Technological progress justifies the evolution of a job from a job occupied by a young to a job occupied by a senior. The requirements to perform a certain task necessarily change over a 20 years period. As in (Mortensen, 1998), firms can create jobs with different levels of productivity depending on their initial and costly investment in specific human capital.

Workers search for a job while unemployed and employed. The arrival frequency of job offers are \( \lambda_i^0 \) for the unemployed and \( \lambda_i \) for the employed. On the firms’ side, firms
receive applicants from unemployment with the frequency $q^0_i$ and from employment with the frequency $q_i$. The mobility of workers depends therefore on both their age and status. These frequencies result from a matching process given the number of vacancies in each market. There are job destruction shocks - each employed worker is displaced into unemployment according to a Poisson process with parameter $s > 0$. All agents are risk neutral.

We assume there exists an institutional minimum wage, $w^*$, in this economy which bounds below the wage distribution. Workers with a working experience are eligible to unemployment benefits depending on the wage they had in their previous job as it exists in France. In this case, workers receive benefits composed of a fixed component $\text{all}$ and a progressive one $\rho$ as follows:

$$b(w) = \rho \times w + \text{all}$$  

These benefits are financed by a lump tax noted $\tau$ that all workers whether employed or unemployed pay.

### 2.2 The workers’ behavior

The asset values of being employed at a wage $w$ are noted $V^e_i(w)$ and solve in each age class:

$$rV^e_i(w) = w - \tau + \lambda_y \int_w^\infty (V^e_y(x) - V^e_y(w))dF_y(x) - s(V^e_y(w) - V^u_y(b(w))) - p(V^e_y(w) - V^e_y(w))$$

$$rV^a_i(w) = w - \tau + \lambda_a \int_w^\infty (V^a_w(x) - V^a_w(w))dF_a(x) - s(V^a_w(w) - V^u_a(b(w))) - p(V^a_w(w) - V^a_w(w))$$

$$rV^s_i(w) = w - \tau + \lambda_s \int_w^\infty (V^s_w(x) - V^s_w(w))dF_s(x) - s(V^s_w(w) - V^u_s(b(w))) - p(V^s_w(w) - V^s_w(w))$$

We denote by $\tau$ the actualization rate. The expected reward for being employed at a wage $w$ is first composed by the wage flow $w$ net of lump taxes $\tau$. Then if the worker meets a firm offering a wage above $w$, he resigns and earns in addition the difference between his current asset value and the value associated to this new wage. The cumulative distribution function of wage offered by firms is noted $F_i(w)$ and the contact probability of employed workers, $\lambda_i$. With the frequency $s$, his job is destroyed and he looses the difference between his current asset value and the asset value of being unemployed noted $V^u_i$. Eventually with the probability $p$, the worker changes age class: if he is young, he becomes seniors, if he is senior, he retires. In this case, he earns or loses the difference between the asset value of being employed at the wage $w$ of the two age classes. We note $V_r$ the asset value of being retired. This value does not depend on the wage $w$.

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4The asset value of being unemployed will be proved to be always below the asset value of being employed later in this subsection.
Given these asset values, it is straightforward for all age classes that high paid jobs last longer than low paid jobs: the resigning probability is a decreasing function of the wage. Thanks to job to job mobilities, workers achieve therefore to select among best paid jobs by resigning from low paid jobs. Note that the higher the contact probability $\lambda_i$ is, the more effective is this selection.

The asset values of unemployed workers who receive the benefit $b$ are noted $V_i^u(b)$ and are given by:

$$rV_y^u(b) = b - \tau + \lambda_0 \int_{R_y(b)} V_y^u(x) - V_y^u(b(w)) dF_y(x) - p(V_y^u(b) - V_y^u(b))$$

$$rV_a^u(b) = b - \tau + \lambda_0 \int_{R_a(b)} V_a^u(x) - V_a^u(b(w)) dF_a(x) - p(V_a^u(b) - V_a^u(b))$$

$$rV_s^u(b) = b - \tau + \lambda_0 \int_{R_s(b)} V_s^u(x) - V_s^u(b(w)) dF_s(x) - p(V_s^u(b) - V_s)$$

The expected reward for being unemployed is first composed by the flow of unemployment benefit. Then, if the worker meets a firm offering a wage above $R_s$, his current asset value and the value associated to being employed at this new wage.

The contact probability of unemployed workers is noted $\lambda_0$. Eventually, as employed workers, with the probability $p_i$, the worker changes age class, and earns or loses the difference between the asset value of unemployed workers of the two age classes.

We can deduce the lowest acceptable wage for a worker receiving the benefits $b$ by setting for each age class of the workers $V_i^u(b) = V_i^e(R_i)$. The level of this reservation wage is therefore given for each age class by:

$$R_y(b) = b + (\lambda_0 - \lambda_y) \int_{R_y} V_y^u(x) - V_y^u(R_y) dF_y(x) + \int_{R_y} p(V_y^u(b) - V_y^u(b(R_y))) + p(V_y^u(b) - V_y^u(R_y))$$

$$R_a(b) = b + (\lambda_0 - \lambda_a) \int_{R_a} V_a^u(x) - V_a^u(R_a) dF_a(x) + \int_{R_a} p(V_a^u(b) - V_a^u(b(R_a))) + p(V_a^u(b) - V_a^u(R_a))$$

$$R_s(b) = b + (\lambda_0 - \lambda_s) \int_{R_s} V_s^u(x) - V_s^u(R_s) dF_s(x) + \int_{R_s} p(V_s^u(b) - V_s^u(b(R_s))) + p(V_s^u(b) - V_s^u(R_s))$$

As in (Chéron and Langot, 2010), the workers’ reservation wages raise with the level of unemployment benefit $b$. The unemployment duration raises with the level of unemployment benefits, and therefore with the previous wage earned by the worker. This indexation of the unemployment benefits on the previous wage is not neutral in function of the workers’ age: workers start their working lives as unemployed with no experience and therefore low unemployment benefits, and over their lives, with working experience, acquire rights to the unemployment insurance system. The way indexed unemployment benefits affect the wage game over the life cycle corresponds to the institutional channel in this paper.
The second term of these reservation wages shows that the workers also take into account the difference of opportunity between the status of unemployed and employed $\lambda^0_i - \lambda_i$ to set their reservation wage. If for example the number of opportunities is higher for unemployed workers $\phi_0 > \phi$, the worker will increase his reservation wage: the accepted wage must compensate this loss. This implication of the contact rate heterogeneity between the two status on the workers’ reservation wage is discussed in (Burdett and Mortensen, 1998). The third term accounts for the fact that the worker anticipates a possible job destruction and the loss it would generate $V^e_i(R_i) - V^u_i(b(R_i))$. Even if $\phi_0 = \phi$, workers will reject a wage equal to $b$ since this new wage will generate benefits lower than $b$, in the case of a job loss. This effect is discussed in (Chéron and Langot, 2010). At last the young and the adult workers take into account the value of being employed in the next period to set their reservation wage of the current period. If for example an adult worker knows that seniors’ reservation wage is higher than the adults’ one ($R_s > R_a$), he will anticipate that on a long run the status of employed is less valuable and will be more reluctant to accept a job as an adult: his reservation wage will increase. Workers’ reservation wages of each age class are therefore co-dependent. As the reservation wage of workers can differ from an age class to another, when workers change age classes, the contract can possibly be broken: it happens when the wage received by the worker in his previous age period is lower than the reservation wage of his current age period. Note that we assume here that the probability to retire does not depend on the workers’ status$^5$ nor on the wage earned.

2.3 The firms’ behavior

The expected profit that firms maximize on each market is given by:

$$
\Pi_y(w, k) = h_y(w)(J_y(w, k) - \beta_y k)
$$
$$
\Pi_a(w, k) = h_a(w)(J_a(w, k) - \beta_a k)
$$
$$
\Pi_s(w, k) = h_s(w)(J_s(w, k) - \beta_s k)
$$

Firms offering a wage $w$ hire a worker of the age class $i$ at the frequency $h_i(w)$. Once the worker hired, the firms get the expected surplus $J_i(w, k)$ of a job of quality $k$ net of the training cost induced by this quality, represented by $\beta_i$.

The firms’ expected profit depends on the labor supply via their hiring frequency. The hiring frequency that the firms face on each market depends on the repartition of workers according to their reservation wage whether they are employed or unemployed. The cumulative distribution function of wage earned by employed workers is noted

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$^5$In reality unemployed workers retire earlier than employed workers. (Hairault, Langot, and Zylberberg, 2012) discuss this issue
$G_i(w)$. The cumulative distribution of benefits earned by unemployed workers is noted $U_i(b)$. On each market, the hiring frequency is therefore given by:

$$h_y(w) = q_y U_y(R_y^{-1}(w)) + q_y (m - u_y) G_y(w) \quad (2)$$
$$h_a(w) = q_a U_a(R_a^{-1}(w)) + q_a (m - u_a) G_a(w) \quad (3)$$
$$h_s(w) = q_s U_s(R_s^{-1}(w)) + q_s (m - u_s) G_s(w) \quad (4)$$

At the frequency $q_i^0$ (resp. $q_i$), a firm contacts an unemployed (resp. employed) worker ready to accept a wage greater than $w$. With a higher wage, firms can in the same time poach a greater number of workers and hire a greater number of indemnified unemployed workers.

The expected surplus induced by employing a worker of each age class at a given wage is given by:

$$J_y(w, k) = \frac{y_y(k) - w + p J_a(w, k)}{r + p + s + \lambda_y(1 - F_y(w))} \quad (5)$$
$$J_a(w, k) = \frac{y_a(k) - w + p J_s(w, k)}{r + p + s + \lambda_a(1 - F_a(w))} \quad (6)$$
$$J_s(w, k) = \frac{y_s(k) - w}{r + p + s + \lambda_s(1 - F_s(w))} \quad (7)$$

With the production function $y_i(k)$, given by:

$$y_y(k) = y_y + \left( \frac{q}{\alpha} \right) k^\alpha$$
$$y_a(k) = y_a + \left( \frac{q}{\alpha} \right) k^\alpha$$
$$y_s(k) = y_s + \left( \frac{q}{\alpha} \right) k^\alpha \quad (8)$$

The parameters $q$ and $\alpha$ are strictly positive exogenous parameters and $y_i$, the workers’ productivity at the workplace, can depend on the age class of the worker. The expected surplus is composed of the margin of the match and of its duration. The margin of the match evolves with the age of the worker, and therefore the match’s productivity function changes over the match’s duration. If the firms hire a young, the match’s productivity is first: $y_y + \left( \frac{q}{\alpha} \right) k^\alpha$, then it becomes: $y_a + \left( \frac{q}{\alpha} \right) k^\alpha$, and eventually is: $y_s + \left( \frac{q}{\alpha} \right) k^\alpha$. The evolution of the workers’ productivity $y_i$ is the first component of what we call the **human capital channel** in this paper.

**Property 1.** The contribution of the workers’ productivity at the workplace to the firms’ profit increases with the wage offered by the firms.

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6Naturally, this progression or regression occurs if the job is not destroyed before the worker changes age class.
Proof. The contribution of the workers’ productivity at the workplace to the firms’ profit can be represented by the derivative of the profit according to this productivity.

\[
\frac{\partial \Pi_y (w, k)}{\partial y} = h_y (w) \frac{\partial J_y (w, k)}{\partial y}
\]

\[
\frac{\partial \Pi_y (w, k)}{\partial y} = \frac{h_y (w)}{r + p + s + \lambda_y (1 - F_y (w))}
\]

\[
\frac{\partial \Pi_a (w, k)}{\partial y_a} = h_a (w) \frac{\partial J_a (w, k)}{\partial y_a}
\]

\[
\frac{\partial \Pi_a (w, k)}{\partial y_a} = \frac{h_a (w)}{r + p + s + \lambda_a (1 - F_a (w))}
\]

\[
\frac{\partial \Pi_s (w, k)}{\partial y_s} = h_s (w) \frac{\partial J_s (w, k)}{\partial y_s}
\]

\[
\frac{\partial \Pi_s (w, k)}{\partial y_s} = \frac{h_s (w)}{r + p + s + \lambda_s (1 - F_s (w))}
\]

These three derivatives raise with the wage offered since \( h_i (w) \) raises with the wage proposed.

The surplus depends positively on the level of specific human capital of the job, \( k \).

To acquire this level, firms must pay a training cost. The marginal cost of training, \( \beta_i \), can be different according to the workers’ age. The decision of firms of the level of specific human capital of the job is the result of the trade-off between the cost of creating a productive job and the return of it in terms of productivity. The result of this trade-off is age-dependant and is given by:

\[
k_s (w) = \left( \frac{\alpha q D_s (w)}{\beta_s} \right)^{\frac{1}{1 - \alpha}}
\]

\[
k_a (w) = \left( \frac{\alpha q D_a (w)}{\beta_a} \right)^{\frac{1}{1 - \alpha}}
\]

\[
k_y (w) = \left( \frac{\alpha q D_y (w)}{\beta_y} \right)^{\frac{1}{1 - \alpha}}
\]

The calculation details are presented in the appendix A, page 37.
The term $D_i$ denotes the discounted expected job duration whether the firm employs a young, an adult or a senior, which is given by:

\[
D_s(w) = \frac{1}{r + p + s + \lambda_s(1 - F_s(w))}
\]

\[
D_a(w) = \frac{1}{r + p + s + \lambda_a(1 - F_a(w))} \left( 1 + \frac{1}{r + p + s + \lambda_s(1 - F_s(w))} \right)
\]

\[
D_y(w) = \frac{1}{r + p + s + \lambda_y(1 - F_y(w))} \left( 1 + \frac{1}{r + p + s + \lambda_a(1 - F_a(w))} \right) \left( 1 + \frac{1}{r + p + s + \lambda_s(1 - F_s(w))} \right)
\]

This trade-off is represented in equations 9, 10 and 11, by the ratio of the investment return (numerator) and the cost of training (denominator). The decision of the jobs’ level of specific human capital depends on age by two aspects:

1. The job duration which raises this investment return ($D_i(w)$)
2. The training cost ($\beta_i$)

The evolution of the cost of specific human capital investment $\beta_i$ is the second component of the human capital channel in this paper. The lower $\beta_i$ is, the higher the level of human capital of workers of the age class $i$ is. The job duration depends negatively on the exogenous job destruction $s$ and positively on the wage paid to the worker $w$, since the frequency at which the worker finds a better offer $\lambda(1 - F_i(w))$ decrease with the wage. The expected job duration is different according to the workers’ age class. Note that, at equal $\lambda_i$ and $F_i$, the job duration decreases with workers’ age class. By affecting the job duration, the horizon affects the expected surplus from employing a worker. The total expected surplus therefore evolves over the job tenure.

3 Equilibrium

The model equilibrium is reached when four distributions and the labor market tightness are stationary on the three markets in the same time:

1. The distribution of wage offered by the firms $F_i$ (Presentation in section 3.1).
2. The distribution of specific human capital investments offered by the firms $k_i$ (Presentation in section 3.1).
3. The distribution of wage received by employed workers $G_i$ (the workers’ flows allowing to compute these distributions are given in appendix C, page 39).
4. The distribution of benefits received by unemployed workers $U_i$ (the workers’ flows allowing to compute these distributions are given in appendix B, page 37).
5. The labor market tightness $\theta_i$, depending on both the composition of workers and the number of vacancies on the market. (Presentation in section 3.2).

In a wage posting game, the firms’ behavior is central. In this model, firms choose, which market to enter, the wage posted and the amount of specific human capital invested.

### 3.1 Wage maximization of profit

In this section, we present how firms choose the posted wages. The distribution of specific human capital investments is directly deduced from these wages since it only depends on the wage offered distribution (see equations 9, 10 and 11). To better understand the firms’ wage game, we can assume firms enter successively on a given market. When there is only one firm on the market, its maximum instantaneous profit will be obtained when it posts the lowest wage of the market. When the other firms enter the market, the intuition is that one firm would have necessarily interest to offer a wage slightly superior to the other to be able to poach all the employed workers. Eventually, (Burdett and Mortensen, 1998) show that at equilibrium, the result of this wage game is a distribution of wage and all the firms reach the same profit. Indeed, when firms increase their offer, their surplus decreases, yet as $F_i(w)$ increases, so their hiring frequency and expected job duration. As $F_i(w)$ cannot be superior to 1, there exists in each market a $w_i$ above which firms have no interest to post wages. Firms therefore spread their wage offers along a wage interval. This maximum wage offered by these firms is computed in order to insure the equiprofit with the firms offering the lowest wage of each market, that we note $\overline{w_i}$. The wage $\overline{w_i}$ therefore solves on each market:

$$
\Pi_y(w_y) = \Pi_y(\overline{w_y})
$$

$$
\Pi_a(w_a) = \Pi_a(\overline{w_a})
$$

$$
\Pi_s(w_s) = \Pi_s(\overline{w_s})
$$

(15)

As the profit is different and can evolve differently from one market to an other with wage, it is likely that the maximum wage would be different on the two markets. In equation 15, note that we also assume that the lowest wage on each market $\overline{w_y}$, $\overline{w_a}$ and $\overline{w_s}$ can be different. Without any regulation on the minimum wage, the lowest wage offered by firms on each market is the wage which maximizes the profit when $F_i(w) = 0$, since it is the lowest wage proposed in the economy. These wages can be
computed as it follows:

\[
\begin{align*}
    w_y &= \arg\max_w \Pi_y(w) \\
    w_a &= \arg\max_w \Pi_a(w) \\
    w_s &= \arg\max_w \Pi_s(w)
\end{align*}
\]

with \( \Pi_y \), \( \Pi_a \) and \( \Pi_s \) the profit of firms offering the lowest wage on each market. Note that if the institutional minimum wage \( w \) is above \( w_y, w_a \) and \( w_s \), then, we can rewrite equation 15 with \( w_y, w_a \) and \( w_s \) equal to \( w \). Eventually, on each age segment, firms spread their wage offer out in order to insure the equiprofit. The distribution of the wages offered by the firms on the youth’s market \( F_y \) solves, from \( w_y \) to \( w_y \):

\[ \Pi_y(w_y) = \Pi_y(w) \] (17)

The distribution of the wages offered by the firms on the adults’ market \( F_a \) solves, from \( w_a \) to \( w_a \):

\[ \Pi_a(w) = \Pi_a(w) \] (18)

And the distribution of the wages offered by the firms on the seniors’ market \( F_s \) solves, from \( w_s \) to \( w_s \):

\[ \Pi_s(w) = \Pi_s(w) \] (19)

### 3.2 Number of vacancies

Firms can move freely from one market to another. As the profit of firms on each market is likely to be different, the number of firms on each market is not the same. The number of firms in each market affects the probability of contact between firms and workers. The number of matches between workers and firms for each age class is indeed given by:

\[
M_i = \phi v_i^\eta (u_i + R_\phi (m - u_i))^{1-\eta}
\]

with \( \eta \) the elasticity of this matching function, \( v_i \) the number of vacancies, \( u_i \) the number of unemployed workers, \( m - u_i \) the number of employed workers and \( R_\phi \) the ratio of the search effectiveness of employed workers \( \phi \) and of the unemployed workers \( \phi^0 \). If we consider that the unemployed workers search more intensively than the employed workers, this ratio will be for instance inferior to 1.

We set \( \theta_i = \frac{v_i}{u_i + R_\phi (m - u_i)} \), the labor market tightness on each market.

The mobility rate of workers of each age class depends on the frequency for a worker to find a job whether he is employed or unemployed. We can express this frequency in
function of the labor market tightness. The frequencies at which an employed and an unemployed worker has a contact with a firm are given by:

\[ \lambda_i = \phi \theta_i^{1-\eta} \]
\[ \lambda_0 = \phi^0 \theta_i^{1-\eta} \]

The frequencies at which a firm has a contact with an employed and an unemployed worker are given by:

\[ q_i = \phi \theta_i^{-\eta} \]
\[ q_0 = \phi^0 \theta_i^{-\eta} \]

At equilibrium, firms enter each market until the profit in each market is equal to the cost of a vacancy noted \( c \). We therefore compute the value of \( \theta_y, \theta_a, \theta_s \), such that:

\[ \Pi_y(w_y, \theta_y) = \Pi_a(w_a, \theta_a) = \Pi_s(w_s, \theta_s) = c \] (20)

**Property 2.** When we assume the institutional minimum wage \( w \) is such that \( w > w_y \), \( w > w_a \) and \( w > w_s \), the level of equiprofit of firms on each market decreases with the labor market tightness of the market.

*Proof.* See appendix D, page 40.

**Corollary.** When workers are ex ante equally productive, the workers’ mobility decreases with workers’ age.

*Proof.* At equal productivity, and mobility, the profit decreases with age.

4 Data, calibration and validation of the model

We present in this section the data we use to calibrate and validate the theoretical model. Then we describe the target used for the calibration and the chosen validation criteria.

4.1 The data source

We use the data of the French Labor Force Survey (Enquête Emploi) of 2002 to calibrate the model. Conducted by the INSEE (Institut National de la Statistique et des Etudes Economiques) since 1950, the French Labor Force Survey provides data such as professions, earnings, and working hours. It is conducted yearly in March on 150 000 people living in 75 000 households. In 2003, the survey evolved and became quarterly,

\[ ^7 \text{This assumption allows a greater simplicity in the calculation} \]
some extra questions were also added. We use the data of this survey just before this change. In this paper, we use the monthly wage after deduction including bonuses (spread monthly). We exclude self employed workers and focus on male wage-earner workers. We choose to restrict attention on a rather homogenous group of workers in terms of educational attainment since in our theoretical framework, workers start their working life with the same productivity and the same level of human capital (they are all ex ante homogenous). The dispersion generated by the heterogeneity of the ex ante workers’ productivity cannot therefore be captured by the model\(^8\). We choose to focus on workers whose educational attainment is the high school degree or considered as “below” (BEP and the CAP) \(^9\) because they constitute the largest group in the sample. We restrict on full-time and part-time workers, and exclude workers with variable hours contracts.

According to Eurostat data computed by the DARES in the report of (Lerais and Marioni, 2004), between 2001 and 2003, the average retirement age in France was slightly inferior to 59 years old (58.8) for an average labor market entry age of 20 years old. We define as in the theoretical model, three age classes evenly long: the 20 to 32 years old, the 33 to 45 years old, and the 46 to 58 years old. We therefore focus on workers between 20 and 58 years old. We compare the wage of these three age classes in cross section. If the real economy were in steady state as in the theoretical economy described in the model, the cross section approach would cause no problem, yet as it is naturally not the case, this approach can show its limits. Notably, by studying the wage of different age classes at a time \(t\), we mix the notion of age and of generation. Therefore, the wage of an age class depends also of the education level of the generation. For instance, workers between 46 and 58 years old in 2002 have the education standards of the sixties. One could argue that to avoid this critic, we should follow a cohort of workers over their life cycle. Yet, this approach supposes that we compare wages at periods where the institutional environment is different. Changes in institutions on the labor market can affect the wage setting decision of firms and the actual wages of workers. We therefore choose the cross section approach. Besides, restricting our study to the workers whose educational attainment is close, protects us partly from the bias generated by the cross section approach. The remaining bias generated by this choice could be a slight underestimation of the trend of human capital accumulation of workers over their life.

The restriction to workers with the same educational attainment causes however another problem: the number of observations decreases significantly with workers’ age.

\(^8\)This limit could be overcome by assuming an exogenous distribution of the ex-ante productivity of young workers: \(y_u\).

\(^9\)Workers whose educational attainment is the high school degree represent too few observations
Indeed, the number of high school graduates in the 90’s is higher than in the 60’s. All aggregated moments computed with the data are therefore biased by this composition. For instance, we have twice as many observations for young workers as for senior workers. In order to make realistic comparison between the results computed on the data and the ones computed based on the model, we compute the model aggregated results so that the proportion of each population within the total population is similar to the one found of the data.

The data on the mean 2002 job to job transition and tenure come from the same Labor Force Survey and were reported by (Lemoine and Wasmer, 2010). The figures of life-cycle job to job transition are computed between 1996 and 1999 by the DARES and are based on data from INSEE. These figures are presented by (Lainé, 2004).

4.2 The data presentation

We deduce hourly wages from the monthly earnings and the hours worked by workers. From these wage data, we draw two wage distributions expressed in French minimum wage: the aggregated wage distribution and the wage distribution according to age class presented in figure 4.2 and 1. It is difficult to pretend that the model developed in the first section can reproduce the extreme wages existing in a wage distribution since in this model workers are ex ante homogenous when they arrive on the labor market. We therefore calibrate this model on a wage distribution corresponding to the first 95 percentiles of the wage distribution of each age.

4.3 The calibration

We set the model period to be one year. The annual interest rate \( r \) is set to 4% as it is usual in the literature. In the data, we assume three life periods of 13 years, we therefore set the probability to change age class to \( \frac{1}{13} \). We normalize the institutional minimum wage since all we expressed all wages in minimum wage. The cost of training for young workers, \( \beta_y \), is also normalized since only the difference between \( \beta_y, \beta_a \) and \( \beta_s \) matters here. We set the elasticity of the matching function to 0.7 as estimated recently by Borowczyk-Martins, Jolivet, Postel-Vinay, (2011). The unemployment benefits are composed of two components, two parameters therefore need to be set: \( \alpha \) and \( \rho \). The fixed component is the unemployment benefits received by the workers with no working experience, it therefore stands for the minimal unemployment benefits that a worker can receive. In France, this minimal income is indexed on the minimum wage and represents

\[ \text{computed by Amossé (2003)} \]
\[ \text{computed by Vandenbrande et al. (2007)} \]
\[ \text{New Estimates of the Matching Function, Working Paper} \]
Figure 2: Aggregated wage distribution for French salaried men (First 95%)

about one third of it. We therefore set \( \alpha = 0.33 \). The progressive parameter of indexation on wage is calibrated in order to reproduce the French unemployment rate. Indeed, if we set this parameter to its institutional level of 57.4\%, we overestimate the level of unemployment benefits in our economy since in the model workers never lose their eligibility to unemployment benefits. In order to reproduce the right value of being unemployed which induce the right level of job rejection in the economy, this parameter should therefore be below this institutional level. According to our calibration results, this parameter is equal to 0.49. Note that using the two components of the unemployment benefits induces that low paid workers receive a higher part of their wage than high paid workers when they become unemployed. This is consistent with the actual French unemployment benefit system.

The other parameters are calibrated on the data presented above. The exogenous destruction rate is calibrated so that to reproduce the median job tenure of 7 years. Our calibration supposes therefore that jobs are exogenously destroyed on average every 14 years. The matching process efficiency parameters for unemployed and employed workers are calibrated respectively on the unemployment duration of 1.14 years and on the average job to job transition of 9\%. The parameters of the production function are calibrated on moments of the wage distribution. The parameter \( q \) is set in order to reproduce the mean wage, \( \alpha \), the median wage, and the parameter of human capital.

\[ ^{13} \text{In 2012, the daily minimum allocation was the gross amount of 28.21 euros (3.16 euros net and hourly) for a hourly net minimum wage of 9.4 euros} \]
accumulation $\beta_a$ and $\beta_s$, the ratio between the 75th centile and the median and the 90th centile and the 75th centile. These last two moments allow to capture the shape of the wage distribution of the second half of the distribution since specific human capital investment allows to explain a great part of wages at the top of the distribution. The value of the return to capital $\alpha$ is close to the value obtained by (Chéron and Langot, 2010) (0.76 instead of 0.77). The workers’ productivity at the workplace $y_y$, $y_a$ and $y_s$, are set in order to reproduce the mode of the respective wage distribution. Our calibration suggests that the cost of training decreases over the workers’ life cycle, yet the actual productivity on the workplace of workers tends to increase at first, between the first two periods, and then decrease for senior workers. Table 1 sums up the annual value of the parameters and the targets used to calibrate them.

<table>
<thead>
<tr>
<th>Table 1: Calibration parameters</th>
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<tbody>
<tr>
<td>Fixed and institutional parameters</td>
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<td>--------------------------------</td>
</tr>
<tr>
<td>$r$</td>
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<tr>
<td>$p$</td>
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<tr>
<td>$\beta_y$</td>
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<tr>
<td>$w$</td>
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<tr>
<td>$a_l$</td>
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<tr>
<td>$\eta$</td>
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<td></td>
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<tr>
<td>Calibrated parameters</td>
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<tr>
<td>$s$</td>
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<tr>
<td>$\phi_0$</td>
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<tr>
<td>$\phi_1$</td>
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<td>$q$</td>
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<td>$\alpha$</td>
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<td>$y_y$</td>
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<td>$y_a$</td>
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<tr>
<td>$y_s$</td>
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<tr>
<td>$\beta_a$</td>
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<tr>
<td>$\beta_s$</td>
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<tr>
<td>$\rho$</td>
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</table>

Note that no calibration on these data is possible without the two components of the workers’ productivity presented in the theoretical model: the evolution of the workers’ productivity at the workplace and the evolution of the training costs. Indeed in a specification without one of these parameters, there is a conflict between an accurate wage dispersion, the wage distribution shape and the increasing path of wage with age. Indeed, the observed wage dispersion and shape can only be obtained by assuming that firms can have different productivities. Yet firms are naturally induced to create lower quality jobs to seniors because of their shorter working horizon. In order to fit the
data, it is therefore necessary to assume the accumulation of human capital of workers. The workers’ productivity at the workplace accounts for a part of the translation of the wage distribution with age, it therefore needs to be included too.

### 4.4 Validation of the model

Figure 3: Simulated distribution of wage, and of wage and match productivity according to workers’ age class—Simulation 1B: French benchmark economy

The simulation induced by this calibration is the **Simulation 1B: French benchmark economy**. The aggregated wage distribution and the wage and productivity distribution over the three age classes generated by this simulation are presented figure 3. From now one, the wage or benefits distributions are in fact densities. We note the densities of $F_i$, $G_i$, and $U_i$, respectively $f_i$, $g_i$ and $u_i$. Table 2 presents the ability of the model to reproduce the evolution of the main moments targeted over the three life periods. We have not searched to reproduce these new moments while calibrating the model, we therefore propose to use them to validate the model. The aggregated
moments used for the calibration are notified in the table in bold letters.

<table>
<thead>
<tr>
<th>Table 2: Validation Results</th>
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<tr>
<td>Total</td>
</tr>
<tr>
<td>Model</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Coefficient of dispersion</td>
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<tr>
<td>Job to job transition</td>
</tr>
<tr>
<td>Unemployment duration</td>
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<tr>
<td>Unemployment rate</td>
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</tbody>
</table>

This model allows to reproduce these new moments quite well and their trend over the three age periods. Note that the wage dispersion remains underestimated since the simulated wage distributions stop earlier than in reality. There again, the model shows its limits in explaining the very top of the distribution. The trend of unemployment rate and unemployment duration follows the data, at the exception of the unemployment rate of seniors which seems overestimated by the model. Yet it is well known that some seniors are declared as retired when they actually are only unable to find a job. It is difficult to assess the number of inactive people who should be considered as unemployed at the end of the life-cycle in the data, yet it is obvious that the empirical unemployment rate among seniors is largely underestimated.

Thanks to the calibration and the simulation of the model presented in this paper, we can infer distributions that are not easily observed in reality and use them to understand the evolution of wage with age. We therefore compute the distribution of wage and productivity offered to the workers by the firms on each market. They are observable on figure 4 (the figures are given by table 8 in appendix F.1, page 49). On the wage distributions (aggregated and over life cycle), we can observe a step close to the wage 2.5. This step actually appears on the simulated distribution of adults, as we can see it on the second graph in figure 3. Looking at the first graph in figure 4 which represents the wage offered to workers according to their age, we can easily explain this phenomenon. Indeed, contrary to the workers employed at a wage no greater than 2.5, all adult workers employed at a wage above 3.5 have necessarily been hired as an adult since young workers are not offered such wages. The adults’wage distribution is therefore composed of a report of young’s wage distribution only up to the wage 3.5. The step that we observe on the simulated distribution of seniors after this same wage is the report of this same step. We observe the same phenomenon at the very top of the distribution of the seniors’wage, around 3, for the exact same reason, as adults are no longer offered such high wages. Naturally, these discontinuities would fade away if we increased the number of age classes. Figure 4 also shows the distribution of unemployed workers according to their previous wage. This distribution
seems to follow the wage distribution of employed workers up to a certain wage, and then increases sharply. At the bottom of this distribution, unemployed workers have low unemployment benefits, there is therefore no job rejection from unemployed workers. The number of workers receiving each level of unemployment benefits only depends on the number of job destruction in the economy. As these destructions do not depend on the workers’ wage, the two distributions have the same shape. Yet above a certain level of unemployment benefits, workers start to reject job offers and to therefore remain longer unemployed. The tail of the unemployed distribution according to their previous wage highlights the fact that the higher the unemployment benefits, the longer the unemployment duration.

Figure 4: Simulated distribution of offered wage, offered productivity, wage and offered job quality- Simulation 1B: French Benchmark economy
5 Results

5.1 Decomposition method into three wage progression channels and results’ summary

We take into account three channels to explain the wage progression over the three age classes: the wage game channel, the human capital channel and the institutional channel. What we call the wage game channel is the evolution of the wage game with age at identical ex ante productivity in a "laissez faire" economy. This channel represents the pure effect of age on the wage game. The human capital channel accounts for the share of the wage progression induced by the productivity increase with age. In the model, the workers’ productivity can evolve by two aspects: first the workers’ productivity at the workplace can change over time and second they can accumulate (or lose) human capital. The first component of this productivity channel affects the match productivity directly by affecting the production function, the second affects it by decreasing the cost of specific human capital for firms. As figure 3 suggests it, the productivity of the match is strongly correlated to the wage of workers. We explain in this section, by which mechanisms the match’s productivity interacts with the workers’ wage. The institutional channel corresponds to the share of the wage progression induced by the presence of the French wage-indexed unemployment insurance system.

To distinguish the contribution of the three channels to the wage progression over life, we need to run two new simulations. The first one is the French benchmark economy in a "laissez faire" economy, i.e. the minimum wage is at the U. S. level, there is no unemployment benefits, and the working life lasts 45 years: Simulation 2B: French benchmark economy without unemployment benefits ($\rho = 0$). In difference with simulation 1B, it allows to assess the institutional channel. Then we simulate the same economy as 2B yet when workers are ex ante evenly productive: Simulation 3B: French benchmark economy with ex ante evenly productive workers and no institution ($y = y_a = y_s = 1.3$ and $\beta_y = \beta_a = \beta_s = 1$), and $\rho = 0$). This simulation allows us to study the wage game. And, in difference with simulation 2B, it allows us to assess the human capital channel. Table 3 and figure 5 display the results of this decomposition.

The effect of institutions is positive on the wage progression. On the 1.48% of yearly wage progression that workers experience in the first part of their working life, 0.10 points of percentage is due to the presence of institutions on the labor market. On the 1% of yearly wage increase that workers experience in the second part of their working life, 0.25 points of percentage is due to their presence. From this table, we can deduce that the wage progression in the first part of the working life is generated by both the productivity channel and the wage game channel, yet to a least extent for this
latter. At the end of the working life, the wage game channel contributes negatively to the wage progression. The next two subsections are devoted to explain these results.

<table>
<thead>
<tr>
<th>Table 3: Decomposition of wage evolution</th>
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<tr>
<td><strong>Monopsony economy</strong></td>
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<tr>
<td>Mean wage</td>
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<td>1</td>
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<td>0%</td>
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<tr>
<td><strong>With on the job search and endogenous productivity (3B)</strong></td>
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<tr>
<td>Mean wage ( g_i )</td>
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<td>1.6</td>
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<td>1.64</td>
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<td>1.52</td>
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<tr>
<td>0.19%</td>
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<td>-0.56%</td>
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<tr>
<td><strong>With evolution of workers' ex ante productivity (2B)</strong></td>
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<tr>
<td>Mean wage ( g_i )</td>
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<td>1.56</td>
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<td>1.84</td>
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<td>2.02</td>
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<td>1.38%</td>
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<td><strong>With institutions (1B)</strong></td>
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<td>Mean wage ( g_i )</td>
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<td>1.61</td>
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<td>1.92</td>
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<td>2.17</td>
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<td>1.48%</td>
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<td>1%</td>
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<tr>
<td><strong>Decomposition of wage progression</strong></td>
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<tr>
<td>Wage game channel per year</td>
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<tr>
<td>0.19 points of %</td>
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<tr>
<td>-0.56 points of %</td>
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<tr>
<td>Productivity channel per year</td>
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<tr>
<td>1.19 points of %</td>
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<tr>
<td>1.32 points of %</td>
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<tr>
<td>Institutional channel per year</td>
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<tr>
<td>0.10 points of %</td>
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<tr>
<td>0.25 points of %</td>
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</table>

5.2 Two mechanisms at stake

The effect of these three channels goes through two of the model mechanisms: the evolution of the workers’ market power and the search effect. If workers had no market power induced by their possibility to search on the job, whatever the productivity of the workers is, firms would have the entire monopsony power on the labor market and would offer to all workers the monopsony wage, i.e. here the minimum wage equal to 1. The possibility of on the job search of workers forces firms to compete to get a share of the workforce and to keep it. In this competition game, the firms can choose the wage to offer and the productivity of the job they want to create. This competition game naturally induces firms to raise their wages and to invest in jobs to increase their productivity. The workers’ capacity to force firms to raise their wage offers (and therefore to create high quality jobs) corresponds to the workers’ market power evolution. Productivity and institutions can affect this market power. The extent of it can be captured by the levels of wages offered by the firms.

The possibility of on the job search allows workers over their life cycle to progressively climb the wage ladder at a given offered wage distribution by resigning from low paid jobs to be employed by high paid jobs. Facing a wage offer lottery, given job to job transitions and after a certain time on the labor market, workers must be able to select themselves among the best paid jobs\(^{14}\). We call this workers’ behavior the search

\(^{14}\)In our model, this selection is highlighted by the shorter tenure of low paid jobs
Figure 5: Decomposition of wage evolution: simulation 1B (benchmark), simulation 2B (wage game and productivity) and simulation 3B (wage game)
effect. In our model this selection over the life cycle can be captured by comparing the gap between the distributions $f_i$ of wage offered to the workers with the distribution $g_i$ of wage received by the workers. The first graph in figure 5 allows this comparison. The wage distributions $g_i$ are composed of higher wages than the distributions of offered wages $f_i$. If this gap increases over the workers’ life cycle, then the search effect contributes to wage progression over the working life.

It is important to distinguish the channels from the mechanisms described above. The mechanisms are the ways the channels affect the wage distribution, when the channels correspond to the source of the wage progression. For that matter, it is possible to cut a channel of wage progression, like by assuming there is no increase in productivity, or no wage game (it is therefore the case of the pure monopsony). It is not the case for the described mechanisms since they constitute the model itself.

5.3 The wage game channel (Simulation 3B)

This subsection explains this evolution by analyzing the forces which rule the wage game and how they evolve over the working life. We start by explaining the evolution of the wage game by the evolution of the workers’ market power and then by the effect of the search.

5.3.1 The workers’ market power over the life-cycle

The comparison of the mean wage offered to workers of each age class ($f_i$) in simulation 3B on the first graph in figure 5 informs us on the evolution of the workers’ market power evolution when they are ex ante evenly productive. The market power of workers in such an economy, is stationary during the first half of the life-cycle and decreasing during the second half of it. When workers enter the labor market, they are unemployed and have not yet had time to raise the wage ladder thanks to job to job mobilities. Over their life, employment situation of workers becomes more favorable: workers integrate into firms and experience ascendant mobility. This backward effect improves the reservation wage of older workers and raise their market power. Yet, aging also means having a shorter working horizon. In a wage game where firms can decide of both the wage and the productivity of the match, the investment dimension of wage is major: workers’ retention allows to avoid extra searching costs for firms and the initial human capital investment to pay for itself. A short horizon makes firms reluctant to compete with other firms for workers’ retention and therefore to offer high wages and create high quality jobs. This forward effect compensates exactly the backward effect in the first part of the working life, yet dominates in the second part as workers get closer

\[\text{Note that in this wage game, workers can have ex post different productivities.}\]
to retirement. The distribution of job qualities presented by figure 6 shows clearly that

without the productivity channel, the firms would be reluctant to create high quality jobs on the seniors’ market, and with a least amplitude on the adults’ market. This is translated in terms of wage by the absence of high wage offers to seniors as the offered wage distribution shows it in figure 6. The distribution of received wage in figure 6 shows that eventually in this context senior workers earning high wages are only those hired in previous life periods.

5.3.2 The search effect

The first graph in figure 5 shows that in the case of the simulation 3B, the search effect hardly contributes to wage progression in the first part of the life cycle and does not contribute at all to this progression in the second part (the gap between $g_i$ and $f_i$ raises slowly and then remains stationary). Intuitively, the selection of good jobs

Figure 6: Simulated distribution of offered wage, offered productivity, wage and offered job quality- Simulation 3B: Wage game
should raise with age. Indeed, the more time workers spend on the labor market, the
more ascendant mobilities they can experience. In the wages adult workers earn for
instance, there is a part inherited from the first life period during which workers have
already had time to select their job. The gap between the wage offered to workers
and the wage received by workers should a priori widens. This occurs lightly in the
first part of the life cycle, yet not at all in the second part. This paradox has two
explanations. First, job openings decrease over the life cycle: the expected surplus
of firms progressively decreases with workers’ age since workers’ horizon gets shorter.
Fewer firms are created on the market of workers closer from their horizon and the job
to job transition rate decreases. Second, as shown by the figure 5 in the simulation 3B,
the dispersion of offered wage lowers with age. When wage dispersion is low, more firms
offer similar wages and the gain from the search is weak. Even if workers can select
among best paid jobs all over their life, as the wage offer lottery they face evolves, this
selection can be totally hampered and workers, after a certain age, have increasingly
more difficulties to find better opportunities. At the bottom line, firms do not even try
to poach employed workers on this market since high wages cannot be amortized on
the long run, they choose instead low wage strategies which target senior unemployed
workers arriving though exogenous destruction. The seniors’ market is a two-speed
market: already employed workers earn rather high wages even if they cannot progress,
while unemployed workers can only find low paid jobs.

Result 1 (Wage game channel). The wage game evolution contributes very lightly
to the wage progression in the first half of the working life and contributes negatively
to this progression in the second half. The finite horizon of workers penalizes greatly
the seniors who are employed by the lowest paid jobs and in the lowest productive job.
Without any improvement of the workers’ productivity over the life-cycle, the wage
would stop increasing at mid-life.

5.4 The human capital channel (Comparison simulation 2B-3B)

According to our results, over their life cycle, accumulation of human capital dominates
its depreciation. The human capital channel accounts for the share of wage progression
induced by the human capital accumulation.

5.4.1 The evolution of the workers’ market power in presence of the human
capital channel

In the last subsection, we have seen that when workers are all ex ante evenly productive,
seniors were penalized in terms of wage offered mostly because firms were reluctant to
create high quality jobs with workers so close to their horizon. The accumulation of
human capital allows to ease the creation of high quality jobs from firms on seniors’ market since due to their high level of human capital seniors need less training to reach the same productivity than workers of the other age classes. The increase in workers’ specific productivity induces firms to compete more intensively in order to hire and retain highly productive workers. In figure 5, the increase in the mean wage and mean quality of jobs offered to adult and senior workers between simulation 3B and 2B shows that the productivity increase over the life cycle allows workers’ market power to raise with age.

5.4.2 The search effect in presence of the human capital channel

Figure 5 shows that the gap between the mean wage and the mean offered wage in simulation 2B is now increasing with age. Thanks to the human capital channel, workers can select effectively among best paid jobs all over their working life. The productivity channel reinforces, or activates (in the second part of the working life) the search channel by increasing the occurrence of job to job mobilities and by inducing firms to offer higher wages (increase in the offered wage dispersion). Note that the widespread offered wage distribution raises the gaps between the wage of two jobs, and therefore without necessarily increasing the occurrence of mobility, increases the gain from mobility.

Result 2 (Human capital accumulation channel). The human capital channel explains for a quite large part the wage progression over the life cycle. Seniors are employed by high quality jobs for two reasons. First because due to their higher level of human capital, firms are induced to create high quality jobs, and second because the greater dispersion of the offered wage (associated to these high quality jobs), and the more frequent job to job transitions allows them to raise their gain from the selecting among the best paid jobs.

5.5 Institutional channel (Comparison simulation 1B-2B)

5.5.1 Effect on workers’ market power

The first graph of figure 5 shows that the unemployment insurance system affects more significantly wage offered to older workers and therefore the older workers’ market power. Young workers, when they enter the labor market, are entitled to the minimum benefits. Over their working life, thanks to working experience, they acquire rights to the unemployment insurance system. As the wage globally raises with workers’ age, the adults are entitled to higher unemployment benefits than the young, and the seniors

\footnote{Yet, we can think that if workers had a mobility cost, the shift in gain from mobility could in this case affect the occurrence of mobility}
Figure 7: Simulated distribution of offered wage, offered productivity, wage and offered job quality- Simulation 2B: French Benchmark economy without UB ($\rho = 0$)
higher than the adults. Therefore, when the minimum wage is high, the young’s unemployment benefits-induced reservation wages are likely to be below minimum wage.

On the adults’ and the senior’s market on the contrary, higher benefits induce reservation wages above minimum wage. On these last two markets, firms’ behavior need to change in order to avoid too many job rejections. The firms’ behavior change on adults’ and seniors’ market can be observed in figure 4 and 7, by comparing the distribution of wage and of offered wage of simulations 1B and 2B. The unemployment insurance system prevents firms from offering low paid jobs, since they would be exposed to high rate of job rejection. They therefore concentrate their wage offer. If wages are close to one another, wage offers close to this mode are more likely to be accepted since workers’ reservation wages respect \( b(w) < w \). In other terms, the unemployment insurance system causes no job rejection without wage dispersion. The wage dispersion decrease induced by the unemployment insurance system occurs therefore mostly at the bottom of the wage distribution.

The non homogenous effect of the unemployment insurance system on workers’ market power over the life cycle necessarily affects wage progression. Table 4 displays the contribution of the unemployment insurance system to wage offer progression in the line ”Progression due to UIS - Wage offer”. Thanks to the unemployment insurance system, wage offer progression is reinforced by 0.65 points of percentage in the first part of the working life and by 0.47 points of percentage in the second part.

5.5.2 Effect on the search effect

Table 4 displays the contribution of the unemployment insurance system to wage progression in the line ”Progression due to UIS - Wage”. Its contribution to wage progression is positive but smaller than on the wage offer progression. Thanks to the unemployment insurance system, wage progression is reinforced by 0.10 points of percentage in the first part of the working life and by 0.25 points of percentage in the second part. At equilibrium, the gap between offered wages and workers’ wages depends on both the intensity and the gain of the workers’ search. However, with an unemployment insurance system, this gap can also be explained by the job selection of unemployed workers (since they reject the less paid ones). The unemployment insurance system has therefore a priori ambiguous effects on job selection since it decreases both the intensity and the gain of the workers’ search. First the unemployment insurance system decreases the firms’ profit, fewer vacancies are created and the workers’ mobility rate decreases. Second, as wage offers are more concentrated, the workers’ mobility at given job to job mobility rate generates lower wage gain. The first graph in figure 5 and the table 4 show that the gap between mean offered wage and the mean earned wage on the adults and seniors’ market is significantly reduced by the unemployment insur-
The effect of job selection of unemployed workers is therefore dominated by the decrease of the search and of the search-induced wage gain. Table 4 gives the extent of this job selection in the economy with and without unemployment benefits. In an economy without unemployment benefits, adults and seniors raise their wage by on average 20.13% and 31.76% by moving from selecting among good jobs, they raise it by only 10.34% and 14.03% in an economy with unemployment benefits. Table 4 also computes the wage progression explained by this mechanism (job selection) in the two economies. When, without unemployment benefits, this selection could contribute to the average wage progression by 0.23 points of percentage in the first part of the working life and 0.5 points of percentage in the second part, in an economy with unemployment benefits, they contribute negatively to the wage progression in the first part of the working life, and only by 0.28 points of percentage in the second part.

Table 4: Selection of good jobs in the economy without unemployment benefits (simulation 4B) and with unemployment benefits (simulation 1B). Effect by age and contribution to the wage progression. Contribution of unemployment insurance system (UIS) to wage and wage offer progression

<table>
<thead>
<tr>
<th></th>
<th>Young</th>
<th>Adults</th>
<th>Seniors</th>
<th>Evolution per year</th>
<th>Evolution per year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$Y \to A$</td>
<td>$A \to S$</td>
</tr>
<tr>
<td><strong>Simulation 4B</strong>: French benchmark without unemployment benefits ($\rho = 0$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean wage ($g_i$)</td>
<td>1.56</td>
<td>1.84</td>
<td>2.02</td>
<td>1.38%$^{¥}$</td>
<td>0.75%</td>
</tr>
<tr>
<td>Mean wage offer ($f_i$)</td>
<td>1.34</td>
<td>1.54</td>
<td>1.59</td>
<td>1.15%</td>
<td>0.25%</td>
</tr>
<tr>
<td>Selection by age - 2B</td>
<td>15.67% $^*$</td>
<td>20.13%</td>
<td>31.76%</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Simulation 1B</strong>: French benchmark economy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean wage ($g_i$)</td>
<td>1.61</td>
<td>1.92</td>
<td>2.17</td>
<td>1.48%</td>
<td>1%</td>
</tr>
<tr>
<td>Mean wage offer ($f_i$)</td>
<td>1.4</td>
<td>1.74</td>
<td>1.9</td>
<td>1.85%</td>
<td>0.72%</td>
</tr>
<tr>
<td>Selection by age -1B</td>
<td>14.75%</td>
<td>10.34%</td>
<td>14.03%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Progression due to selection of good jobs

- Progression due to selection of good jobs - 4B: 0.23$^{¥}$ points of %
- Progression due to selection of good jobs - 1B: -0.37 points of %

Progression due to UIS

- Progression due to UIS - Wage offer: 0.65$^{¥}$ points of %
- Progression due to UIS - Wage: 0.10 points of %

$^*$ Young workers achieve to raise their wage by 15.67% by selecting into best jobs in economy 4B ($1.56^{15.67\%} - 1$)

$^{¥}$ Over the first half of the working life, mean wage raises by 1.38% per year in economy 4B ($1.38\%\times\frac{1}{2}$)

† On the 1.38% of yearly wage increase over the first half of the working life, 0.23 points are induced by the selection of good jobs in economy 4B ($1.38\%\times\frac{1}{2} - 1$)

‡ On the 1.85% of yearly wage offer increase over the first half of the working life, 0.65 points are induced by the UIS in economy 1B ($1.85\% - 1.15\%$)
5.5.3 Effect of unemployment benefits in the U.S. (simulation 2A)

Do unemployment benefits have the same effect on all kinds of labor market? To answer this question, we simulate the U.S. economy with the same level of unemployment benefits as there exists in France: Simulation 2A: U.S. benchmark economy with unemployment benefits ($\rho = 0.49$ and $\text{all} = 0.33$) and compare it with the U.S. Benchmark: Simulation 1A: U.S. benchmark economy ($\rho = 0$). The U.S. data, the calibration and validation of the model on these data, and the results of these two simulations are presented in appendix E, page 44. Using the figure 10\(^{17}\) From these results, we easily notice that the effect of the same unemployment insurance system would be stronger on the U.S. economy: the unemployment insurance system would induce a wage increase of on average 7.69% (all age classes mixed) instead of 2.19% in France, an unemployment duration would increase by 515% instead of by 221% and an unemployment rate would increase by 144% instead of by 89%. This stronger effect of the unemployment insurance system goes in particular through a more significant effect on the youth’s labor market. The global higher effect of unemployment benefits comes first from the higher level of wage dispersion in the U.S. This dispersion is induced on the top of the distribution by a higher level of job to job mobility and at the bottom of the wage distribution by the lower minimum wage (20% lower). When wages are more spread out, so are the unemployment benefits and more job rejections occur. As a low minimum wage allows a higher dispersion at the bottom of the wage distribution in particular on the young’s market, this effect is stronger on their market. Besides, in the U.S., employment to unemployment and unemployment to employment mobilities are structurally more frequent: $s = 0.072$ in France and $s = 0.12$ in the U.S. and the average unemployment duration without unemployment benefits are 0.5 years in France and 0.23 years in the U.S.. U.S. workers therefore go more frequently through unemployment periods in their life and the reservation wage of unemployed workers affects in greater proportion the labor market.

Result 3 (Unemployment insurance system). In spite of its negative effect on the intensity and the gain of the workers’ search, the unemployment insurance system reinforces wage progression thanks to its increasing effect on the workers’ market power: it contributes to wage progression by 0.10 points of percentage in the first part of the working life and by 0.25 points of percentage in the second part. Figure 5 shows that the unemployment insurance system partly accounts for the lower mobility of French workers, but not entirely. Thanks to this insurance system, wage progression can occur without high level of workers’ mobility, in particular in the second part of the work-

\(^{17}\)Note that figure 10 is expressed in French minimum wage for the values of mean wage and mean offered wage in order to ease the comparison with the French case (figure 5).
Note that the elasticity of major labor market outcomes to the replacement rate depends greatly on the initial structure of the labor market: the same unemployment insurance system has far stronger effect in an economy like the U.S. where wage dispersion and job destruction rate are greater.
6 Conclusion

This paper allows to assess the life cycle effect of a major French institution. A significant share of wage progression in France is fueled by the unemployment insurance system. Besides, this insurance system also accounts for a large decrease of search-induced wage progression, via its negative effect on both mobility rate and wage dispersion. The presence of unemployment benefits partly accounts for the lower mobility of French workers, yet not entirely. The unemployment insurance-induced wage contraction could partly explain the remaining gap between the job to job mobility rate in France and in the U. S.. In our framework, workers change jobs as soon as they receive an offer above their current wage, the gap between the two wages does not matter, only does the order of the wages. In this context, the wage dispersion affects weakly the occurrence of mobility (only when firms post the exact same wage). In reality, mobility is costly for workers, and they do not change jobs unless the wage gap between the two jobs compensates this cost. Adding mobility cost on the workers’ side could reinforce the negative effect of the unemployment insurance system on the mobility rate found in this paper. The natural extension of this work is to compute the optimal profile of these unemployment benefits (Beveridgian and Bismarckian component). (Mortensen, 1998) shows in its infinite horizon model that because of the firms’ monopsony power to set wages, there are too numerous vacancies at equilibrium. The first best equilibrium can be restored by reducing this power thanks to the implementation of a minimum wage or unemployment benefits. When life cycle is taken into account, a single level of the minimum wage or of the unemployment benefits is not enough to restore this first best equilibrium: one value per age is needed. The wage-indexed unemployment benefits allow to vary the unemployment benefits with the workers’ age. This variation depends on the two components of the unemployment benefits, the Beveridgian and Bismarckian. The aim of this extension would be therefore to define the value of these two components and therefore to compute the optimal profile of the unemployment insurance system allowing the economy to reach its second best equilibrium.
A Decision of the level of specific human capital

The derivative of the profit of firms targeting each age according to the quality of the job is given by:

\[
\frac{\partial \Pi_s(w, k)}{\partial k} = qk^{\alpha-1} - \beta_s(r + p + s + \lambda_s(1 - F_s(w)))
\]

\[
\frac{\partial \Pi_a(w, k)}{\partial k} = qk^{\alpha-1}\left(1 + \frac{p}{r + p + s + \lambda_a(1 - F_a(w))}\right) - \beta_a(r + p + s + \lambda_a(1 - F_a(w)))
\]

\[
\frac{\partial \Pi_y(w, k)}{\partial k} = qk^{\alpha-1}\left(1 + \frac{p}{r + p + s + \lambda_y(1 - F_y(w))}\right) - \beta_y(r + p + s + \lambda_y(1 - F_y(w)))
\]

The second derivative is given by:

\[
\frac{\partial^2 \Pi_s(w, k)}{\partial k^2} = (\alpha - 1)qk^{\alpha-2}
\]

\[
\frac{\partial^2 \Pi_a(w, k)}{\partial k^2} = (\alpha - 1)qk^{\alpha-2}\left(1 + \frac{p}{r + p + s + \lambda_a(1 - F_a(w))}\right)
\]

\[
\frac{\partial^2 \Pi_y(w, k)}{\partial k^2} = (\alpha - 1)qk^{\alpha-2}\left(1 + \frac{p}{r + p + s + \lambda_y(1 - F_y(w))}\right) - \beta_by(r + p + s + \lambda_y(1 - F_y(w)))
\]

When the production function has a decreasing return to the job quality, the second derivatives are negative. The first order condition \(\frac{\partial \Pi_i(w, k)}{\partial k} = 0\), for \(i = y; a; s\) gives the result of equations 11, 10, and 9.

B Unemployed workers’ flows

When the unemployed workers do not receive any unemployment benefits or receive unemployment benefits which do not depend on wage, all the workers of the same age class have the same reservation wage induced by the labor opportunity cost and their allocations if any. In this context, the lowest wage offered by the firms on each market is necessarily greater than this reservation wage, since no firm has interest in offering a wage that no worker can accept. For that matter, in that context, there is no job rejection from unemployed workers.

When workers receive progressive unemployment benefits, wage dispersion generates an heterogeneity of unemployment benefits among workers: unemployed workers have ex post different reservation wages. Firms are therefore exposed to offer wages that can be rejected by some unemployed workers, yet accepted by some others. The only unemployed workers who never reject any wage for sure are those who receive the lowest benefits in the economy: workers with no working experience. In this framework, the reserve army is only constituted of these workers. In steady state, the flows into and out of this reserve army noted \(u_i(all)\) are equal. It therefore solves the following flows


All young workers entering the labor market \((p \cdot m)\) receive these minimum unemployment benefits. Among the adults and the seniors, the workers who receive these minimum benefits are those who have never worked since they enter the labor market, that is the workers who have not yet found a job when adult or senior. The mass of unemployed workers according to their unemployment benefits for all \(b > all\), solves in steady state the following flows equations:

\[
\begin{align*}
[\lambda_0^y + p]u_y(all) &= p \cdot m \\
[\lambda_0^a + p]u_a(all) &= pu_y(all) \\
[\lambda_0^s + p]u_s(all) &= pu_a(all)
\end{align*}
\] (21)

Unemployed workers who receive a benefit \(b\) accept a job if the wage proposal associated to this job is above \(R_i(b)\). With the frequency \(\lambda_i^0[1 - F_i(R_i(b))]\), unemployed workers receiving \(b\) have a contact with a firm offering a wage above his reservation wage, in other terms it is the job finding frequency of unemployed workers receiving the benefits \(b\). Note that \(u_i(b)\) depends both on the density of wage in the economy and on the offered wage density by the firms.

The total unemployment rate of each market is the sum of unemployment rates for each level of unemployment benefit, it depends on the repartition of unemployment benefits in the economy. By using 21 and 22, we can deduce the unemployment rate on each market:
The mass of workers receiving a wage no greater than \( w \) is given for each age class by \((m - u_i)G_i(w)\). In steady state, the flows into and out of firms offering a wage no greater than \( w \) for each age class are equal. They are given by:

\[
(p + s + \lambda_y(1 - F_y(w)))(m - u_y)G_y(w) = \lambda_y \int_w^\infty f_y(x)U_y(R_y^{-1}(x))dx \\
(p + s + \lambda_s(1 - F_s(w)))(m - u_s)G_s(w) = \lambda_s \int_w^\infty f_s(x)U_s(R_s^{-1}(x))dx + p(m - u_s)G_s(w) \tag{23}
\]

\[
(p + s + \lambda_s(1 - F_s(w)))(m - u_s)G_s(w) = \lambda_s \int_w^\infty f_s(x)U_s(R_s^{-1}(x))dx + p(m - u_s)G_s(w)
\]

On the left side of these equations, there is the flow of workers out of firms offering a wage no greater than \( w \). These workers either experience an exogenous shock with the frequency \( s \), change age class with the probability \( p \) or resign to be employed by a higher paying job with the frequency \( \lambda(1 - F_i(w)) \). On the right side there is the flow of workers into firms offering a wage no greater than \( w \). The unemployed workers do not accept a wage unless it is above his reservation wage. The term \( U_i(R_i^{-1}(x)) \) represents the number of unemployed workers of age \( i \) who accept all of offers greater that the wage \( x \), these workers receive benefits which make them reject all the offers lower than \( x \). The term \( p(m - u_y)G_y(w) \) of the left side of equation 24 means that part of the workers are already employed when they become adults. It is not the case for the youth who all start as unemployed. The wage distribution of adults partly depends on the report of the wage distribution of the youth, and similarly for seniors. Note that the distribution of wages in the economy depends on the distribution of unemployment benefits among unemployed workers.
The mass of employed workers earning a wage below or equal to w is given by:

\[
(m - u_y)G_y(w) = \frac{\lambda_g \int_w^w f_y(x)U_y(R_y^{-1}(x))dx}{p + s + \lambda_y(1 - F_y(w))}
\]

\[
(m - u_a)G_a(w) = \frac{\lambda_a \int_w^w f_a(x)U_a(R_a^{-1}(x))dx + p(m - u_y)G_y(w)}{p + s + \lambda_a(1 - F_a(w))}
\]

\[
(m - u_s)G_s(w) = \frac{\lambda_s \int_w^w f_s(x)U_s(R_s^{-1}(x))dx + p(m - u_a)G_a(w)}{p + s + \lambda_s(1 - F_s(w))}
\]

(24)

D  Effect of the labor market tightness on profit

Seniors’ Labor Market  The effect of labor market tightness on the profit on the seniors’ market is given by:

\[
\Pi_s(w, \theta_s) = h_s(w, \theta_s) \left( J_s(w, \theta_s) - \beta_s k_s(w, \theta_s) \right)
\]

\[
\frac{\partial \Pi_s(w, \theta_s)}{\partial \theta_s} = \frac{\partial h_s(w, \theta_s)}{\partial \theta_s} \left( J_s(w, \theta_s) - \beta_s k_s(w, \theta_s) \right) + h_s(w, \theta_s) \left( \frac{\partial J_s(w, \theta_s)}{\partial \theta_s} - \beta_s \frac{\partial k_s(w, \theta_s)}{\partial \theta_s} \right)
\]

With,

\[
h_s(w, \theta_s) = q_s^0 u_s
\]

\[
h_s(w, \theta_s) = \phi^0 \theta_s^{-\eta} \frac{sm + pu_a}{p + s + \phi \theta_s^{1-\eta}}
\]

\[
\Rightarrow \frac{\partial h_s(w, \theta_s)}{\partial \theta_s} < 0
\]

And with,

\[
\frac{\partial J_s(w, \theta_s)}{\partial \theta_s} = \frac{q \frac{\partial k_s(w, \theta_s)}{\partial \theta_s}}{r + p + s + \lambda_s} - \frac{y_s(k_s(w, \theta_s)) - w}{(r + p + s + \lambda_s)^2}
\]

\[
\frac{\partial J_s(w, \theta_s)}{\partial \theta_s} = \frac{\partial k_s(w, \theta_s)}{\partial \theta_s} \left( \frac{q k_s(w, \theta_s)^{\alpha - 1}}{r + p + s + \lambda_s} \right) - \frac{y_s(k_s(w, \theta_s)) - w}{(r + p + s + \lambda_s)^2}
\]
Therefore,

\[
\frac{\partial J_s(w, \theta_s)}{\partial \theta_s} - \beta_s \frac{\partial k_s(w, \theta_s)}{\partial \theta_s} = \frac{\partial k_s(w, \theta_s)}{\partial \theta_s} \left(\frac{qk_s(w, \theta_s)^{\alpha-1}}{r + p + s + \lambda_s} - \beta_s\right)
\]

= 0 by definition of \(k_s\)

\[
= y_s(k_s(w, \theta_s)) - \frac{w}{(r + p + s + \lambda_s)^2}
\]

\[
\Rightarrow \frac{\partial J_s(w, \theta_s)}{\partial \theta_s} - \beta_s \frac{\partial k_s(w, \theta_s)}{\partial \theta_s} < 0
\]

Consequently, \(\frac{\partial \Pi_a(w, \theta_a)}{\partial \theta_a} < 0\).

**Adults’ Labor Market** We proceed as well on the adults’ market. The effect of labor market tightness on the profit on the adults’ market is given by:

\[
\frac{\partial \Pi_a(w, \theta_a)}{\partial \theta_a} = h_a(w, \theta_a) (J_a(w, \theta_a) - \beta_a k_a(w, \theta_a))
\]

\[
= \frac{\partial h_a(w, \theta_a)}{\partial \theta_a} (J_a(w, \theta_a) - \beta_a k_a(w, \theta_a))
\]

\[
+ h_a(w, \theta_a) \left(\frac{\partial J_a(w, \theta_a)}{\partial \theta_a} - \beta_a \frac{\partial k_a(w, \theta_a)}{\partial \theta_a}\right)
\]

With,

\[
h_a(w, \theta_a) = q_0^a u_a
\]

\[
h_a(w, \theta_a) = \phi^a \theta_a^{-\eta} \frac{sm + pu_a}{p + s + \phi^a \theta_a^{-\eta}}
\]

\[
\Rightarrow \frac{\partial h_a(w, \theta_a)}{\partial \theta_a} < 0
\]
And with after simplifications,

\[
\frac{\partial J_a(w, \theta_a)}{\partial \theta_a} = \frac{q \frac{\partial k_a(w, \theta_a)}{\partial \theta_a}}{r + p + s + \lambda_a} k_a(w, \theta_a)^{\alpha - 1} + p \frac{\partial J_a(w, \theta_a)}{\partial \theta_a} - \frac{y_a(k_a(w, \theta_a)) - w + p J_s(w, \theta_a)}{(r + p + s + \lambda_a)^2}
\]

\[
\frac{\partial J_a(w, \theta_a)}{\partial \theta_a} = \frac{\partial k_a(w, \theta_a)}{\partial \theta_a} \left( \frac{q k_a(w, \theta_a)^{\alpha - 1}}{r + p + s + \lambda_a} + \frac{p}{r + p + s + \lambda_a} \right) \left( r + p + s + \lambda_a - y_a(k_a(w, \theta_a)) - w \right) - \frac{y_a(k_a(w, \theta_a)) - w + p J_s(w, \theta_a)}{(r + p + s + \lambda_a)^2}
\]

Therefore,

\[
\frac{\partial J_a(w, \theta_a)}{\partial \theta_a} - \beta_a \frac{\partial k_a(w, \theta_a)}{\partial \theta_a} = \frac{\partial k_a(w, \theta_a)}{\partial \theta_a} \left( \frac{q k_a(w, \theta_a)^{\alpha - 1}}{r + p + s + \lambda_a} + \frac{p}{r + p + s + \lambda_a} \right) \left( r + p + s + \lambda_a - y_a(k_a(w, \theta_a)) - w \right) - \frac{y_a(k_a(w, \theta_a)) - w + p J_s(w, \theta_a)}{(r + p + s + \lambda_a)^2}
\]

Consequently, \( \frac{\partial \Pi_y(w, \theta_y)}{\partial \theta_y} < 0. \)

**Youth’s Labor Market**  The effect of labor market tightness on the profit on the youth’s market is given by:

\[
\Pi_y(w, \theta_y) = h_y(w, \theta_y) (J_y(w, \theta_y) - \beta_y k_y(w, \theta_y))
\]

\[
\frac{\partial \Pi_y(w, \theta_y)}{\partial \theta_y} = \frac{\partial h_y(w, \theta_y)}{\partial \theta_y} (J_y(w, \theta_y) - \beta_y k_y(w, \theta_y)) + h_y(w, \theta_y) \left( \frac{\partial J_y(w, \theta_y)}{\partial \theta_y} - \beta_y \frac{\partial k_y(w, \theta_y)}{\partial \theta_y} \right)
\]
With,
\[ h_y(w, \theta_y) = q_y^0 u_y \]
\[ h_y(w, \theta_y) = \phi_y^0 \theta_y^{-\eta} \frac{(s + p)m}{p + s + \phi_y^1 \theta_y^{-\eta}} \]
\[ \Rightarrow \frac{\partial h_y(w, \theta_y)}{\partial \theta_y} < 0 \]

And with,
\[ \frac{\partial J_y(w, \theta_y)}{\partial \theta_y} \]
\[ = \frac{\partial k_y(w, \theta_y) k_y(w, \theta_y)^{\alpha - 1}}{r + p + s + \lambda_y} + \frac{\partial J_y(w, \theta_y)}{\partial \theta_y} - \frac{y_y(k_y(w, \theta_y)) - w + p J_y(w, \theta_y)}{(r + p + s + \lambda_y)^2} \]
\[ = \frac{\partial k_y(w, \theta_y) \left(q k_y(w, \theta_y)^{\alpha - 1} + \frac{\partial J_y(w, \theta_y)}{\partial \theta_y}\right)}{r + p + s + \lambda_y} \]
\[ + \frac{p}{r + p + s + \lambda_y} \left(\frac{p}{r + p + s + \lambda_y} \right) \]
\[ \times \left(\frac{q k_y(w, \theta_y)^{\alpha - 1} + \frac{\partial J_y(w, \theta_y)}{\partial \theta_y} - y_y(k_y(w, \theta_y)) - w}{(r + p + s + \lambda_y)^2} \right) \]
\[ - \frac{y_y(k_y(w, \theta_y)) - w + p J_y(w, \theta_y)}{(r + p + s + \lambda_y)^2} \]

After simplifications, we get:
\[ \frac{\partial J_y(w, \theta_y)}{\partial \theta_y} \]
\[ = \frac{\partial k_y(w, \theta_y) \left(q k_y(w, \theta_y)^{\alpha - 1} + \frac{p}{r + p + s + \lambda_y} \right)}{r + p + s + \lambda_y} \]
\[ + \left(\frac{p}{r + p + s + \lambda_y} \right) \left(\frac{p}{r + p + s + \lambda_y} \right) \]
\[ \times \left(\frac{q k_y(w, \theta_y)^{\alpha - 1} + \frac{\partial J_y(w, \theta_y)}{\partial \theta_y} - y_y(k_y(w, \theta_y)) - w}{(r + p + s + \lambda_y)^2} \right) \]
\[ - \frac{y_y(k_y(w, \theta_y)) - w + p J_y(w, \theta_y)}{(r + p + s + \lambda_y)^2} \]
\[ - \frac{p}{r + p + s + \lambda_y} \left(\frac{y_y(k_y(w, \theta_y)) - w + p J_y(w, \theta_y)}{(r + p + s + \lambda_y)^2} \right) \]
\[
\frac{\partial J_y(w, \theta_y)}{\partial \theta_y} = \frac{\partial k_y(w, \theta_y)}{\partial \theta_y} \frac{q}{r + p + s + \lambda_y} \times \left( k_y(w, \theta_y)^{\alpha - 1} + \frac{1}{r + p + s + \lambda_a} \left( pk_y(w, \theta_y)^{\alpha - 1} + \frac{p^2 k_y(w, \theta_y)^{\alpha - 1}}{r + p + s + \lambda_a} \right) \right) \\
- y_y(k_y(w, \theta_y)) - w + p J_a(w, \theta_y) \\
- \frac{p}{r + p + s + \lambda_y} y_y(k_y(w, \theta_y)) - w + p J_a(w, \theta_y) \\
- \frac{p}{r + p + s + \lambda_y} \left( y_y(k_y(w, \theta_y)) - w \right)
\]
\[
= 0 \text{ by definition of } k_y(w)
\]
\[
\Rightarrow \frac{\partial J_y(w, \theta_y)}{\partial \theta_y} - \beta_a \frac{\partial k_a(w, \theta_a)}{\partial \theta_a} < 0
\]

Therefore, we have the following inequality:

\[
\frac{\partial J_y(w, \theta_y)}{\partial \theta_y} < 0
\]

Consequently, \( \frac{\partial k_a(w, \theta_a)}{\partial \theta_y} < 0. \)

**E U.S. calibration**

**E.1 The data**

We use the data of the 2002 Annual Social and Economic Supplement (ASEC) to calibrate the model. The ASEC is an annual report of the statistical Current Population Survey (CPS) conducted monthly by the United States Census Bureau for the Bureau of Labor Statistics. Some supplemental questions are added in the ASEC (in March), notably on income received in the previous calendar year, which are used to estimate the data on income and work experience. The ASEC is split in three records, the household record, the family record and the person record. In this paper, we use the value of monthly earnings before deduction of the longest job over the last calendar year.
year on the person record. We restrict on men, and exclude self-employed workers and only focus on wage-earner workers. As for French data, we choose to restrict attention on an homogenous group of workers in terms of educational attainment. We choose to focus on workers whose educational attainment is the high school degree or equivalent because they constitute the largest group in the sample.

We focus on workers between 20 and 64 years old. Data from OECD show indeed that the actual retirement age in the U. S. was 64 years old in 2002, even if the legal age is fixed to 67 years old. We define as in the theoretical model, three age classes evenly long: the 20\(^{18}\) to 34 years old, the 35 to 49 years old, and the 50 to 64 years old. We therefore focus on workers between 20 and 64 years old.

To calibrate the model, we also use data on transitions within the labor force: the mobility frequency between unemployment and employment and between jobs. We use the transitions rate computed by (Menzio, Telyukova, and Visschers, 2012) over the life-cycle. Their data comes from the U.S. Census’Survey of Income and Program Participation (SIPP) of the 1996 to 2000 period.

Eventually, the data we use for job tenure are 2002 data which come from a survey published by the Bureau of Labor Statistics of the U.S. Department of Labor on job tenure in 2012 and the data of 2002 unemployment rate by age are provided by the online database of the OECD.

We convert gross earnings to net earnings and deduce from this net monthly earnings and the hours worked in the job, the net hourly wage. We express the wages received by workers of the three age classes in federal minimum wages (4.33$ per hour in 2002). Like on the French data, we restrict our analysis on the first 95 percentiles of the wage distribution. The distribution of wage by age class is resent in the figure E.1.

E.2 The calibration

The calibration method is similar to the one describe on French data and is summed up in table 5.

The simulation induced by this calibration is the **Simulation 1A: U. S. benchmark economy**. The simulated aggregated wage distribution, the wage distribution over the three age classes and the distribution of productivity over the three age classes are presented on the figure 9.

As for the French calibration, table 6 presents the ability of the model to reproduce some extra moments on the labor market: unemployment rate and standard dispersion, and the evolution over the three life periods of the main moments targeted.

\(^{18}\)From 20 years old, the employment rate is above 50%, oecdstats
Figure 8: U. S. wage distribution of salaried men (First 95%) expressed in US minimum wage

Table 5: Calibration parameters

<table>
<thead>
<tr>
<th>Fixed and institutional parameters</th>
<th>Targets' value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r ) 0.04</td>
<td>discounted rate</td>
</tr>
<tr>
<td>( p ) 1/15</td>
<td>working life duration</td>
</tr>
<tr>
<td>( \beta_y ) 1</td>
<td>Normalized</td>
</tr>
<tr>
<td>( \eta ) 0.7</td>
<td>fixed</td>
</tr>
<tr>
<td>( \beta_a ) 0.88</td>
<td>C75/C50</td>
</tr>
<tr>
<td>( \beta_s ) 0.659</td>
<td>C90/C75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calibrated parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( s ) 0.12</td>
<td>Median job tenure 3.7 years</td>
</tr>
<tr>
<td>( \phi_0 ) 7.15</td>
<td>Unemployment duration 0.33 years</td>
</tr>
<tr>
<td>( \phi_1 ) 3</td>
<td>Job to job transition 21%</td>
</tr>
<tr>
<td>( q ) 0.425</td>
<td>Mean Wage 2.6</td>
</tr>
<tr>
<td>( \alpha ) 0.72</td>
<td>Median Wage 2.4</td>
</tr>
<tr>
<td>( y_y ) 1.75</td>
<td>Mode of young 1.8</td>
</tr>
<tr>
<td>( y_a ) 2.26</td>
<td>Mode of adults 2.4</td>
</tr>
<tr>
<td>( y_s ) 2.14</td>
<td>Mode of seniors 2.5</td>
</tr>
<tr>
<td>( \beta_a ) 0.88</td>
<td>C75/C50 1.3</td>
</tr>
<tr>
<td>( \beta_s ) 0.659</td>
<td>C90/C75 1.2</td>
</tr>
</tbody>
</table>
Figure 9: Simulated distribution of wage, and of wage and match productivity according to workers’ age class—Simulation 1A: U. S. benchmark economy.
Figure 10: Comparison of the U. S. economy with (simulation 2A) and without unemployment benefits (simulation 1A), expressed in French minimum wage for mean wage and mean offered wage.
Table 6: Validation Results

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Young (20-34)</th>
<th>Adults (35-49)</th>
<th>Senior (50-654)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model</td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>Mean</td>
<td>2.6</td>
<td>2.6</td>
<td>2.2</td>
<td>2.2</td>
</tr>
<tr>
<td>Dispersion Coef</td>
<td>30.8%</td>
<td>38.5%</td>
<td>26.8%</td>
<td>36%</td>
</tr>
<tr>
<td>Job to job transition</td>
<td>0.21</td>
<td>0.21</td>
<td>0.22</td>
<td>0.28</td>
</tr>
<tr>
<td>Unemployment duration</td>
<td>0.33</td>
<td>0.33</td>
<td>0.25</td>
<td>0.27</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>5.5%</td>
<td>5.6%</td>
<td>7%</td>
<td>7.3%</td>
</tr>
</tbody>
</table>

Table 7: Moments of simulated distribution of offered wage, offered productivity, wage and offered job quality - Simulation 1A: Benchmark economy

<table>
<thead>
<tr>
<th></th>
<th>Total Population</th>
<th>Young</th>
<th>Adults</th>
<th>Seniors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean wage ($g_i$)</td>
<td>2.62</td>
<td>2.18</td>
<td>2.82</td>
<td>3</td>
</tr>
<tr>
<td>Mean wage offered ($f_i$)</td>
<td>1.88</td>
<td>1.64</td>
<td>2.04</td>
<td>2.02</td>
</tr>
<tr>
<td>Dispersion of wages ($\sigma(g_i)$)</td>
<td>0.3</td>
<td>0.27</td>
<td>0.27</td>
<td>0.28</td>
</tr>
<tr>
<td>Dispersion of wages offered ($\sigma(f_i)$)</td>
<td>0.39</td>
<td>0.28</td>
<td>0.32</td>
<td>0.36</td>
</tr>
<tr>
<td>Mean new hired productivity ($y_i(k)$)</td>
<td>2.61</td>
<td>2.16</td>
<td>2.86</td>
<td>2.98</td>
</tr>
<tr>
<td>→ With workers’ specific productivity ($y_i$)</td>
<td>2.04</td>
<td>1.75</td>
<td>2.26</td>
<td>2.14</td>
</tr>
<tr>
<td>→ With quality of the job ($qk^\alpha$)</td>
<td>0.57</td>
<td>0.41</td>
<td>0.60</td>
<td>0.84</td>
</tr>
</tbody>
</table>

F French Simulations

F.1 Simulations 1B

Table 8: Moments of simulated distribution of offered wage, offered productivity, wage and offered job quality- Simulation 1B: French Benchmark economy

<table>
<thead>
<tr>
<th></th>
<th>Total Population</th>
<th>Young</th>
<th>Adults</th>
<th>Seniors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean wage ($g_i$)</td>
<td>1.85</td>
<td>1.61</td>
<td>1.92</td>
<td>2.17</td>
</tr>
<tr>
<td>Mean wage offered ($f_i$)</td>
<td>1.68</td>
<td>1.4</td>
<td>1.74</td>
<td>1.9</td>
</tr>
<tr>
<td>Dispersion of wages ($\sigma(g_i)$)</td>
<td>0.22</td>
<td>0.18</td>
<td>0.187</td>
<td>0.19</td>
</tr>
<tr>
<td>Dispersion of wages offered ($\sigma(f_i)$)</td>
<td>0.23</td>
<td>0.16</td>
<td>0.16</td>
<td>0.2</td>
</tr>
<tr>
<td>Mean new hired productivity ($y_i(k)$)</td>
<td>2.32</td>
<td>1.74</td>
<td>2.33</td>
<td>3.13</td>
</tr>
<tr>
<td>→ With workers’ specific productivity ($y_i$)</td>
<td>1.53</td>
<td>1.4</td>
<td>1.61</td>
<td>1.8</td>
</tr>
<tr>
<td>→ Quality of the job ($qk^\alpha$)</td>
<td>0.79</td>
<td>0.34</td>
<td>0.72</td>
<td>1.33</td>
</tr>
</tbody>
</table>

F.2 Simulations 2B
Table 9: Moments of simulated distribution of offered wage, offered productivity, wage and offered job quality- Simulation 2B: French Benchmark economy without unemployment insurance system

<table>
<thead>
<tr>
<th></th>
<th>Total Population</th>
<th>Young</th>
<th>Adults</th>
<th>Seniors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean wage ($g_t$)</td>
<td>1.81</td>
<td>1.56</td>
<td>1.84</td>
<td>2.02</td>
</tr>
<tr>
<td>Mean wage offered ($f_t$)</td>
<td>1.5</td>
<td>1.34</td>
<td>1.54</td>
<td>1.59</td>
</tr>
<tr>
<td>Dispersion of wages ($\sigma(g_t)$)</td>
<td>0.24</td>
<td>0.2</td>
<td>0.217</td>
<td>0.24</td>
</tr>
<tr>
<td>Dispersion of wages offered ($\sigma(f_t)$)</td>
<td>0.27</td>
<td>0.19</td>
<td>0.23</td>
<td>0.28</td>
</tr>
<tr>
<td>Mean new hired productivity ($y_t(k)$)</td>
<td>2.11</td>
<td>1.71</td>
<td>2.14</td>
<td>2.7</td>
</tr>
<tr>
<td>Workers'specific productivity ($y_t$)</td>
<td>1.53</td>
<td>1.4</td>
<td>1.61</td>
<td>1.8</td>
</tr>
<tr>
<td>Quality of the job ($q_{k_t}$)</td>
<td>0.58</td>
<td>0.31</td>
<td>0.53</td>
<td>0.9</td>
</tr>
</tbody>
</table>

References


